Ministry of Education
Young Scholars Club
International Olympiad on Astronomy and Astrophysics
Tehran, Iran, Oct. 17-26, 2009

3rd International Olympiad of Astronomy and Astrophysics

Observational Competition

Personal Information

Name:	Country:
Student Code:	

Please read these instructions carefully:

1. All participants will receive a problem set, a writing board, a pen, a ruler and a headlight by the organizers.

2. This competition consists of two parts:

i) Two questions on “Naked Eye observation”. You Have 12 minutes to answer these two questions.

ii) One question on “Using a telescope”. Each part of this question has a specific time, which is mentioned in your question sheet.

3. All participants will be guided by assistants to the observing site until returning to the waiting hall. Assistants will collect the answer and problem sheets.

4. Do not forget to fill out the boxes at the top of each answer sheet with your country name and your student code.

5. You have 2 minutes to familiarize yourself with Observing ground and darkness of your environment, just before starting the exam time in observing ground.

6. Examiner’s alarm will indicate the beginning and the end of each part of your exam.

7. Each problem has a specific guideline which helps you during the exam.

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Astronomy and Astrophysics
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Observational Competition

Naked Eye Observations

Time: 12 Minutes

You Have 12 minutes to answer the questions of the Naked Eye Observations (Question 1 and Question 2)

Question 1

1.1: Figure 1 (frame size $\cong 100^{\circ} \times 70^{\circ}$) shows a part of the sky, for 22 October 2009 at 21:00 local time. Four bright stars in Perseus and Andromeda constellations are missing in this chart. Find these missing stars by looking at the sky. Then, draw a cross on the location of each missing bright star in these two constellations on the chart (i.e. figure 1). Use numbers in table 1-1 to indicate these crosses**.**

Note : Polaris is indicted by "N" symbol in figure 1.

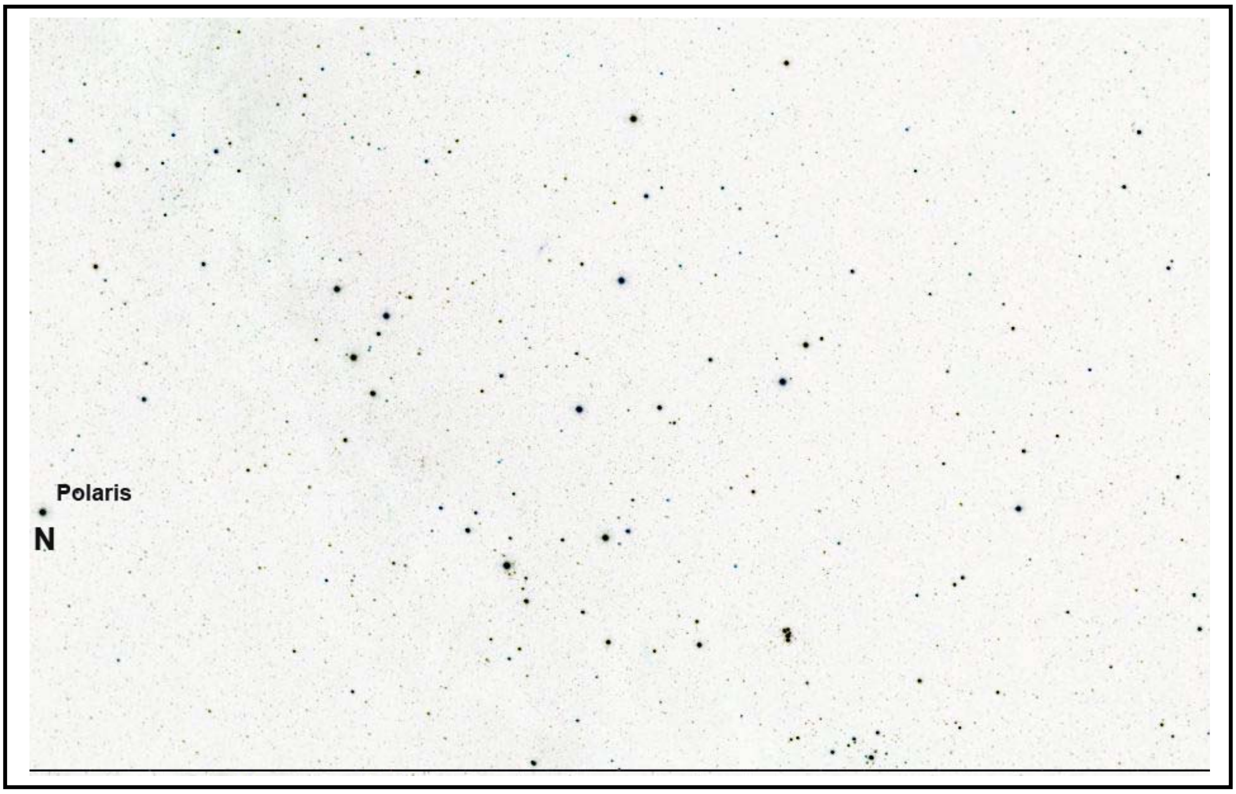
(40 Points)

Table 1-1

Number	Common Name	Bayer Names
1	Mirfak	Alpha Persei
2	Alpheratz	Alpha Andromeda
3	-	Epsilon Persei
4	Menkib	Xi Persei
5	-	Gamma Persei
6	Algol	Beta Persei
7	Almach	Gamma Andromeda
8	-	Delta Andromeda
9	-	51 Andromeda
10	Mirach	Beta Andromeda
11	Atik	Zeta Persei

Name:	Country:
Student Code:	

Question 1 - Figure 1



Question 2

2.1: Figure 2 shows a part of the sky which contains **Cephei constellation**, for 22 October 2009 at 22:00 local time. Five bright stars in Cephei constellation are identified by numbers (1, 2, … , 5) and common names. Estimate the angular distances (in units of degrees) between two pairs of stars shown in table 2-1 and complete this table with your answers. **(40 Points)**

Tables 2-1

Angular Distance	
Pairs of stars	**Angular Distance (degrees)**
1 (Errai) and 2 (Alfirk)	
1 (Errai) and 3 (Alderamin)	

2.2: Use table 2-2 and figure 2, then Estimate the "apparent visual magnitude" of stars 2 (Alfirak) and 3 (Alderamin) and complete Table 2-3. **(40 Points)**

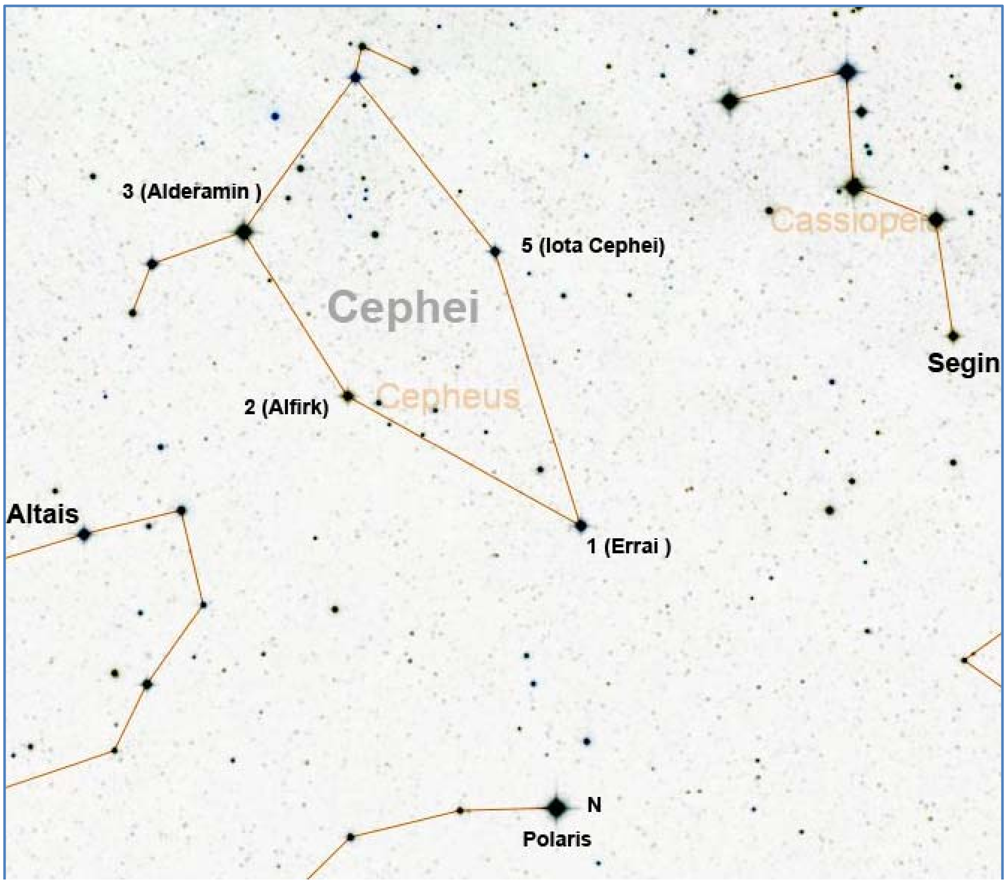
Table 2-2

Star Name	Apparent Visual Magnitude
Polaris	1.95
Altais	3.05
Segin	3.34
All of these stars, are marked in the figure 2	

Table 2-3

Magnitude Estimation		
Star Number	**Star Name**	**Apparent Visual Magnitude**
2	Alfirk	
3	Alderamin	

Question 2 - Figure 2



3rd International Olympiad of Astronomy and Astrophysics

Observational Competition

Telescopic Observations

Time: 13 Minutes

Note: You have only 13 Minutes to answer all parts of this Question.

Name:	Country:
Student Code:	Examiner Code:

Question 3

Before starting this part, please note:

The telescope is pointed by the examiner towards Caph (α Cas). Please note the readings on the grade circles before moving the telescope (to be used in 3.2).

3.1:

Choose one of the 4 recommended stars listed below; write down the name of the selected star in table 3-1 and point the telescope to that star. Then, notify the examiner to check it. **(40 Points)**

1- Deneb (Alpha Cygni)
2- Alfirk (Beta Cephei)
3- Algol (Beta Persei)
4- Capella (Alpha Aurigae)

(You have 6 Minutes to answer 3.1)

Table 3-1

Name of selected star

Name:	Country:
Student Code:	

3.2: The Telescope was parked to Caph in Cassiopeia constellation (RA: 0h:9.7m ; Dec: 59°:12'). Using the clock beside the telescope write down the local time (with the format of HH:MM:SS) in the appropriate field in Table B. Then, by using the graded circle on the telescope mount, estimate the "declination" and the "hour angle" of the target measured from South, which you chose in part one of this question. Then, complete Table 3-2. **(40 Points)**

(You have 7 Minutes to answer 3.2)

Table 3-2

Name and Coordinates of the Selected Star			Local Time :
Name of Selected Star	Hour Angle (hh:mm)	Declination (°:')	

3rd International Olympiad of Astronomy and Astrophysics

Practical Competition

Personal Information

Name:	**Country:**
Student Code:	

Problem 1: CCD Image Processing (60 points)

As an exercise of image processing, this problem involves use of a simple calculator and tabular data (table 1.1) which contains the pixel values of an image during the given exposure time. This image, which is a part of a larger CCD image, was taken by a small CCD camera, installed on an amateur telescope and using a *V* band filter. Figure 1.1 shows this 50×50 pixels image that contains 5 stars.

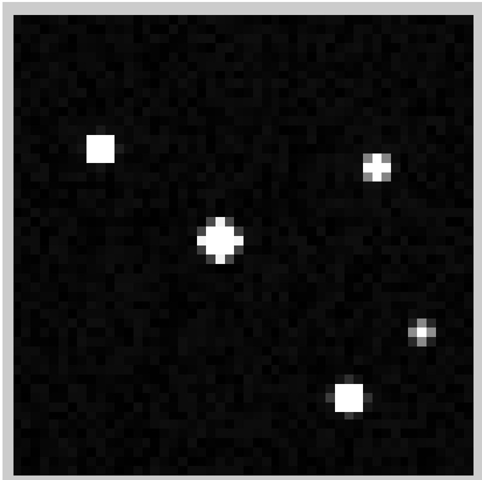


Figure 1.1

In table 1.1 the first row and column indicates the pixels' x and y coordinates. Table 1.2 gives the telescope and the image specifications.

Table 1.2

Telescope focal length	$1.20\ m$
CCD pixel size	$25 \times 25\ \mu m$
Exposure time	$450\ s$
Telescope zenith angle	25°
Average extinction coefficient in V band	0.3 mag/airmass

The observer identified stars 1, 3 and 4 by comparing this image with standard star catalogues. Table 1.3 shows stars true magnitudes (m_t) as given in the catalogue.

Table 1.3

Star	m_t
1	9.03
3	6.22
4	8.02

a. Using the available data, determine the instrumental magnitudes of the stars in the image. Assume the dark current is negligible and the image is flat fielded. For simplicity you can use a square aperture.
Hint: The instrumental magnitude is calculated using the difference between the measured flux from the star in the aperture and the flux from an equivalent area of dark sky.

b. The instrumental magnitude of a star in a CCD image is related to true magnitude as

$$m_I = m_t + KX - Zmag$$

where K is extinction, X is airmass, m_I and m_t are respectively instrumental and true magnitude of star and $Zmag$ is zero point constant. Calculate the zero point constant ($Zmag$) for identified stars. Calculate average zero point constant ($Zmag$).

Hint: Zero point constant is the constant reducing extinction-free magnitudes to the true magnitude.

c. Calculate true magnitudes of stars 2 and 5.

d. Calculate CCD pixel scale for the CCD camera in units of arcsec.

e. Calculate average brightness of dark sky in magnitude per square arcsec (m_{sky}).

f. Use a suitable plot to estimate astronomical seeing in arcsec.

Problem 2: Venus(60 point)

An observer in Deh-Namak (you will be taking the observational part of the exam in this region tonight) has observed Venus for seven months, started from September 2008 and continued until March 2009. During the observation, a research CCD camera and an image processing software were used to take high resolution images and to extract high precision data. Table 2.1 shows the collected data during the observation.

Table 2.1 description:

Column 1	Date of observation.
Column 2	Earth-Sun distance in astronomical unit (AU) for observation date and time. This value is taken from high precision tables.
Column 3	Phase of Venus, Percent of Venus disk illuminated by the Sun as observed from the Earth.
Column 4	Elongation of Venus, the angular distance between center of the Sun and center of Venus in degrees as observed from the Earth.

a) Using given data in table 2.1, calculate the Sun-Venus-Earth angle($\angle SVE$) . This is angular separation between the Sun and the Earth as seen from Venus. Write $\angle SVE$ angle in column 2 of Table 2.2 in your answer sheet for the all observing dates.
Hint: Remember that the line between light and shadow, in the phases, is an ellipse arc.

b) Calculate Sun – Venus distance in AU and write it down in column 3 of table 2.2 for all observation dates.

c) Plot Sun – Venus distance versus observing date.

d) Find perihelion $(r_{v,min})$ and aphelion $(r_{v,max})$ distances of Venus from the Sun.

e) Calculate semi-major axis (a) of the Venus orbit.

f) Calculate eccentricity (e) of Venus orbit.

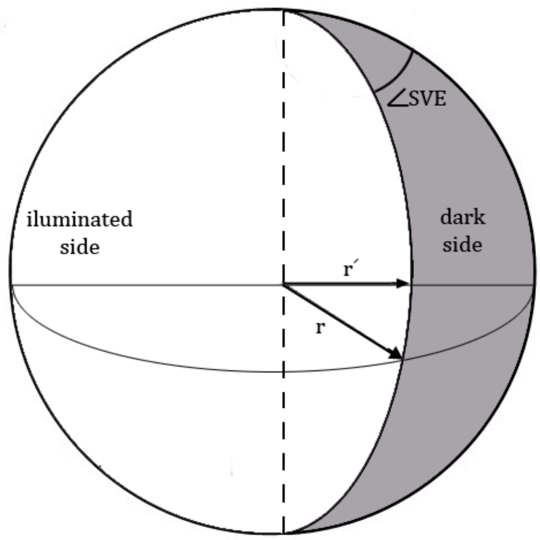


Figure (part a)

Table 2.1

Column 1	Column 2	Column 3	Column 4
Date	Earth - Sun Distance (AU)	Phase (%)	Elongation (SEV) (°)
20/9/2008	1.0043	88.4	27.56
10/10/2008	0.9986	84.0	32.29
20/10/2008	0.9957	81.6	34.53
30/10/2008	0.9931	79.0	36.69
9/11/2008	0.9905	76.3	38.71
19/11/2008	0.9883	73.4	40.62
29/11/2008	0.9864	70.2	42.38
19/12/2008	0.9839	63.1	45.29
29/12/2008	0.9834	59.0	46.32
18/1/2009	0.9838	49.5	47.09
7/2/2009	0.9863	37.2	44.79
17/2/2009	0.9881	29.6	41.59
27/2/2009	0.9904	20.9	36.16
19/3/2009	0.9956	3.8	16.08

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Long Problems

Problem 16: High Altitude Projectile (45 points)

A projectile which initially is put on the surface of the Earth at the sea level is launched with the initial speed of $v_\circ = \sqrt{(GM/R)}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. M and R are the mass and radius of the Earth respectively. Ignore the air resistance and rotation of the Earth.

a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R$.
b) Calculate the highest altitude of the projectile with respect to the Earth surface (in the unit of the Earth radius).
c) What is the range of the projectile (distance between launching point and falling point)?
d) What is eccentricity (e) of the ellipse?
e) Find the flying time for the projectile.

Problem 17: Apparent number density of stars in the Galaxy (45 points)

Let us model the number density of stars in the disk of Milky Way Galaxy with a simple exponential function of $n = n_0 \exp\left(-\frac{r-R_0}{R_d}\right)$, where r represents the distance from the center of the Galaxy, R_0 is the distance of the Sun from the center of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars (red clump) as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

a) Considering a limiting magnitude of $m = 18$ for a telescope, calculate the maximum distance that telescope can detect the red clump stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.
b) Assume an extinction of $0.70\, mag/kpc$ for the interstellar medium. Repeat the calculation as done in the part (a) and obtain a rough number for the maximum distance these red giant stars can be observed.
c) Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute f of overall stars. In this part assume no extinction in the interstellar medium as part (a).
Hint : the Tylor expansion of $y = \log_{10} x$ is :

$$y = y_0 + \frac{1}{ln10} \frac{x - x_0}{x}$$

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Long Problems

Solutions

Solution 16:

a) Total energy of the projectile is

$$E = \frac{1}{2}mv_\circ^2 - \frac{GMm}{R} = -\frac{GMm}{2R} < 0$$

$E < 0$ means that orbit might be ellipse or circle. As $\theta > 0$, the orbit is an ellipse.
Total energy for an ellipse is

$$E = -\frac{GMm}{2a}$$

Then

$$a = R$$

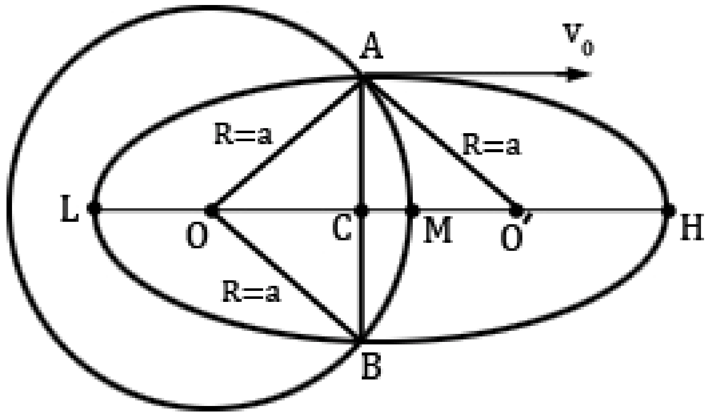


Figure (1)

b) In figure (1) we have

$$OA + O'A = 2a$$

$$O'A = a$$

In OAO' triangle it is obvious that

$$OC = CO'$$

Then C must be the center of the ellipse with the initial velocity vector $v_\circ$ parallel to the ellipse major-axis (LH).

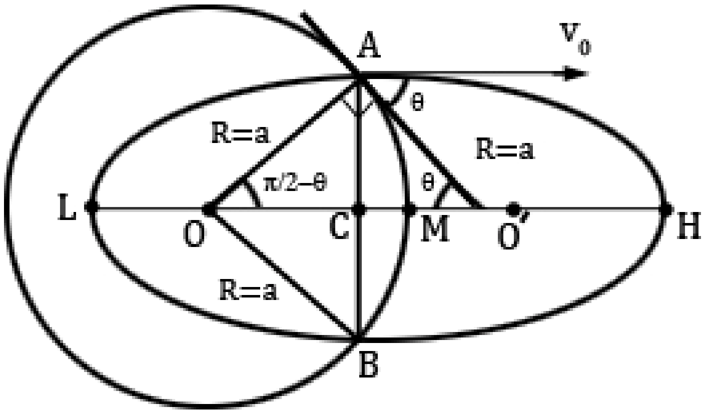


Figure (2)

In figure (2)

$$HM = CH - CM = a - (R - R\sin\theta) = R - R + R\sin\theta = R\sin\theta = \frac{R}{2}$$

c) Range of the projectile is $\widehat{AB}$

$$\widehat{AB} = 2\left(\frac{\pi}{2} - \theta\right)R = (\pi - 2\theta)R = \frac{2\pi}{3}R$$

d) Start with ellipse equation in polar coordinates

$$r = \frac{a(1-e^2)}{1 + ecos\varphi}$$

For point A

$$R = \frac{R(1-e^2)}{1 - ecos(\frac{\pi}{2} + \theta)}$$

$$e = sin\theta = \frac{1}{2}$$

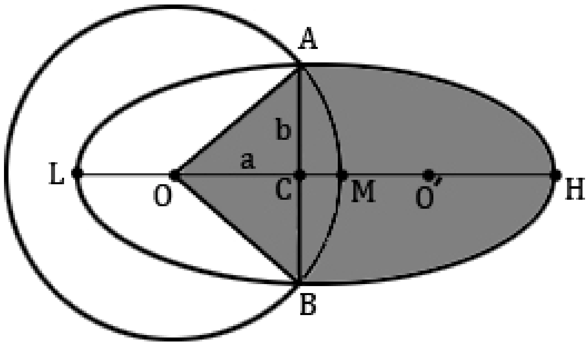


Figure 3

e) Using Kepler's second law

$$\frac{\Delta S}{S_0} = \frac{\Delta T}{T}$$

$$\Delta S = S_{AOBH} = S_{\Delta AOB} + \frac{S_0}{2}$$

$$= 2 \times \frac{\mathrm{bae}}{2} + \frac{\pi \mathrm{ab}}{2} = \mathrm{bae} + \frac{\pi \mathrm{ab}}{2}$$

$$\frac{\Delta S}{S} = \frac{\mathrm{bae} + \frac{\pi \mathrm{ab}}{2}}{\pi ab} = \frac{e + \frac{\pi}{2}}{\pi} = \frac{0.5 + \frac{\pi}{2}}{\pi}$$

Kepler’s third law

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = 84.5\ min$$

$$\Delta T = T\ \times \frac{0.5 + \frac{\pi}{2}}{\pi} = 55.7\ min$$

Solution 17:

a) Relation between the apparent and absolute magnitude is given by

$$m = M + 5log\left(\frac{d}{10}\right)$$

where d is in terms of parsec. Substituting $m = 18$ and $M = -0.2$, results in

$$d = 4.37 \times 10^4\ pc$$

b) Adding the term for the extinction, changes the magnitude distance relation as follows

$$m = M + 0.7x + 5\log(100x)$$

where x is given in terms of kilo parsec. To have a rough value for x, after substituting m and M, this equation reduces to

$$18.2 = 0.7x + 5\log(x)$$

To solve this equation, we examine

$$x = 5\ , 5.5\ , 6\ , 6.5$$

where the best value is obtained roughly $x \cong 6.1\ kpc$.

c) For a solid angle Ω, the number of observed red clump stars at the distance in the range of x and $x + \Delta x$ is given by

$$\Delta N = \Omega\, x^2\, n(x) f\ \Delta x$$

So the number of stars observed in Δx is given by

$$\frac{\Delta N}{\Delta x} = \Omega\, x^2\, n(x) f$$

From the relation between the distance and apparent magnitude we have

$$m_1 = M + 5log\left(\frac{x}{10}\right)$$

$$m_2 = M + 5log\left(\frac{x + \Delta x}{10}\right)$$

$$\Delta m = 5log\left(\frac{x + \Delta x}{x}\right)$$

$$\Delta m = 5log\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10} ln\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10}\left(\frac{\Delta x}{x}\right)$$

Replacing Δx with Δm, results in

$$\frac{\Delta N}{\Delta m} = \frac{\Delta N}{\Delta x} \times \frac{\Delta x}{\Delta m}$$

So the number of stars for a given magnitude is obtained by

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n(x) x^3 f$$

Finally we substituting x in terms of apparent magnitude using $\mathrm{x} = 10^{\frac{\mathrm{m}-9.78}{5}}$.
In the case of no extinction, we are able to observe the Galaxy beyond the center. So $\frac{dN}{dm}$ has two terms in $x < R_0$ and $x > R_0$. The relation between x and r for these two cases are

$$x = R_0 - r \qquad\qquad x < R_0$$

and

$$x = R_0 + r \qquad\qquad x > R_0$$

So in general we can write $\frac{\Delta N}{\Delta m}$ as

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n_0 \exp\left(\frac{10^{\frac{\mathrm{m}-9.78}{5}}}{R_d}\right) \times 10^{\frac{3(m-9.78)}{5}} f \qquad x < R_0$$

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} \mathrm{n}_0 \exp\left(\frac{2\mathrm{R}_0}{R_d}\right) \exp\left(-\frac{10^{\frac{\mathrm{m}-9.78}{5}}}{R_d}\right) \times 10^{\frac{3(m-9.78)}{5}} f\ \Theta(x_0 - x) \quad x > R_0$$

where $\Theta(x)$ is the step function and $x_0 = 44.1\ kpc$ is the maximum observable distance.

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Please read these instructions carefully:

1. Each student will receive problem sheets in English and/or in his/her native language.

2. The available time for answering theoretical problems is 5 hours. You will have 15 short problems (Theoretical Part 1, Problem 1 to 15), and 2 long problems (Theoretical Part 2, Problem 16 and 17).

3. Use only the pen that has been provided on your desk.

4. Do **Not** use the back side of your writing sheets. Write only inside the boxed area.

5. Yellow scratch papers are not considered in marking.

6. Begin answering each problem in separate sheet.

7. Fill in the boxes at the top of each sheet of your paper with your "country name", your "student code", "problem number", and total number of pages which is used to answer to that problem.

8. **Write the final answer for each problem in the box, labeled "Answer Sheet".**

9. Starting and the end of the exam will be announced by ringing a bell.

10. The final answer in each question part must be accompanied by units, which should be in SI or appropriate units. 20% of the marks available for that part will be deducted for a correct answer without units.

11. The required numerical accuracy for the final answer depends on the number of significant figures given in the data values in the problem. 20% of the marks available for the final answer in each question part will be deducted for answers without required accuracy as given in the problem. Use the constant values exactly as given in the table of constants.

12. At the end of the exam put all papers, including scratch papers, inside the envelope and leave everything on your desk.

Table of Constants

(All constants are in SI)

Parameter	Symbol	Value
Gravitational constant	G	
Plank constant	h	6.63×10^{-34} J s
Speed of light	c	
Solar Mass		
Solar radius		
Solar luminosity		
Apparent solar magnitude (V)		-26.8
Solar constant		
Mass of the Earth		
Radius of the Earth		
Mean density of the Earth		$5 \times 10^{3}\ kg\ m^{-3}$
Gravitational acceleration at sea level	g	
Tropical year		365.24 days
Sidereal year		365.26 days
Sidereal day		86164 s
Inclination of the equator with respect to the ecliptic	ε	
Parsec	pc	
Light year		
Astronomical Unit	AU	
Solar distance from the center of the Galaxy		
Hubble constant	H	
Mass of electron		
Mass of proton		
Central wavelength of V-band	λ	
Refraction of star light at horizon		
	π	3.1416

Useful mathematical formula:

3rd International Olympiad on Astronomy and Astrophysics

Theoretical Competition

Short Problems

Short Problems: (10 points each)

Problem 1: Calculate the mean mass density for a super massive black hole with total mass of $1\times10^{8}\ M_{\odot}$ inside the Schwarzschild radius.

Problem 2: Estimate the number of photons per second that arrive on our eye at $\lambda = 550\ nm$ (V-band) from a G2 main sequence star with apparent magnitude of $m = 6$ (the threshold of naked eye visibility). Assume the eye pupil diameter is $6\ mm$ and all the radition from this star is in $\lambda = 550\ nm$.

Problem 3: Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same.

Problem 4: In a typical Persian architecture, on top of south side windows there is a structure called "Tabeshband" (shader), which controls sunlight in summer and winter. In summer when the Sun is high, Tabeshband prevents sunlight to enter rooms and keeps inside cooler. In the modern architecture it is verified that the Tabeshband saves about 20% of energy cost. Figure (1) shows a vertical section of this design at latitude of $36°.0$ N with window and Tabeshband.

Using the parameters given in the figure, calculate the maximum width of the Tabeshband, "x", and maximum height of the window , "h" in such a way that:

i) No direct sunlight can enter to the room in the summer solstice at noon.
ii) The direct sunlight reaches the end of the room (indicated by the point **A** in the figure) in the winter solstice at noon.

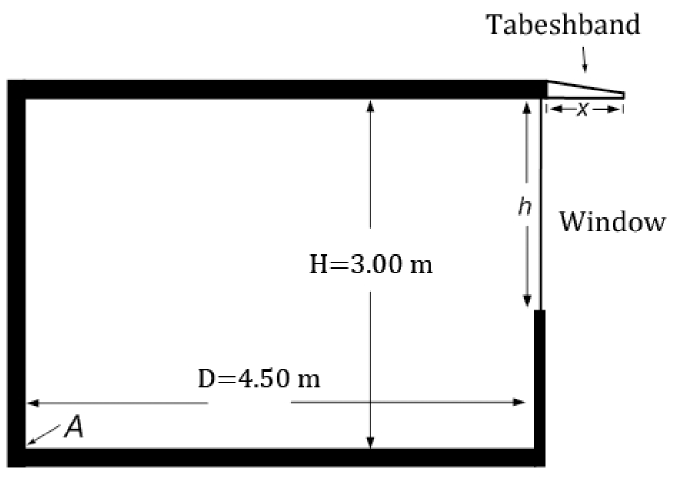


Figure (1)

Problem 5: The Damavand Mountain is located at the North part of Iran, in south coast of Caspian Sea. Consider an observer standing on the Damavand mountaintop (latitude = $35°\,57'$N; longitude = $52°\,6'$ E; altitude $5.6 \times 10^3\ m$ from the mean sea level) and looking at the sky over the Caspian Sea. What is the minimum declination for a star, to be seen marginally circumpolar for this observer. Geodetic radius of the Earth at this latitude is $6370.8\ km$. Surface level of the Caspian Sea is approximately equal to the mean sea level.

Problem 6: Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass of M and radius R. Ignore collisions between the particles of the cloud and the launched object. If the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$.

Problem 7: A student tries to measure field of view (FOV) of the eyepiece of his/her telescope, using rotation of the Earth. To do this job, the observer points the telescope towards Vega (alpha Lyr., RA: 18.5^h , Dec: +39°), turns off its "clock drive" and measures trace out time, t=5.3 minutes, that Vega crosses the full diameter of the FOV. What is the FOV of this telescope in arc-minutes?

Problem 8: Estimate the mass of a globular cluster with the radius of $r = 20\ pc$ and root mean square velocity of stars equal to $v_{rms} = 3\ kms^{-1}$.

Problem 9: The Galactic longitude of a star is $l = 15°$. Its radial velocity with respect to the Sun is $V_r = 100\ kms^{-1}$. Assume stars in the disk of the Galaxy are orbiting the center with a constant velocity of $V_0 = 250\ kms^{-1}$ in circular orbits in the same sense in the galactic plane. Calculate distance of the star from the center of the Galaxy.

Problem 10: A main sequence star with the radius and mass of $R = 4R_\odot$, $M = 6M_\odot$ has an average magnetic field of 1×10^{-4} T. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of $20\ km$.

Problem 11: Assume the mass of neutrinos is $m_\nu = 10^{-5} m_e$. Calculate the number density of neutrinos (n_ν) needed to compensate the dark matter of the universe. Assume the universe is flat and 25 % of its mass is dark matter.

Hint: Take the classical total energy equal to zero

Problem 12: Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermo-nuclear reactions in its center in 100 years. Assume the Earth's orbit remains circular during this period.

Problem 13: Assume that you are living in the time of Copernicus and do not know anything about Kepler's laws. You might calculate Mars-Sun distance in the same way as he did. After accepting the revolutionary belief that all the planets are orbiting around the Sun, not around the Earth, you measure that the orbital period of Mars is 687 days, then you observe that 106 days after opposition of Mars, the planet appears in quadrature. Calculate Mars-Sun distance in astronomical unit (AU).

Problem 14: A satellite is orbiting around the Earth in a circular orbit in the plane of the equator. An observer in Tehran at the latitude of $\varphi = 35.6^{\circ}$ observes that the satellite has a zenith angle of $z = 46.0°$, when it transits the local meridian. Calculate the distance of the satellite from the center of the Earth (in the Earth radius unit).

Problem 15: An eclipsing close binary system consists of two giant stars with the same sizes. As a result of mutual gravitational force, stars are deformed from perfect sphere to the prolate spheroid with $a = 2b$, where a and b are semi-major and semi-minor axes (the major axes are always co-linear). The inclination of the orbital plane to the plane of sky is $90°$. Calculate the amplitude of light variation in magnitude (Δm) as a result of the orbital motion of two stars. Ignore temperature variation due to tidal deformation and limb darkening on the surface of the stars.

Hint: A prolate spheroid is a geometrical shape made by rotating of an ellipse around its major axis, like rugby ball or melon.