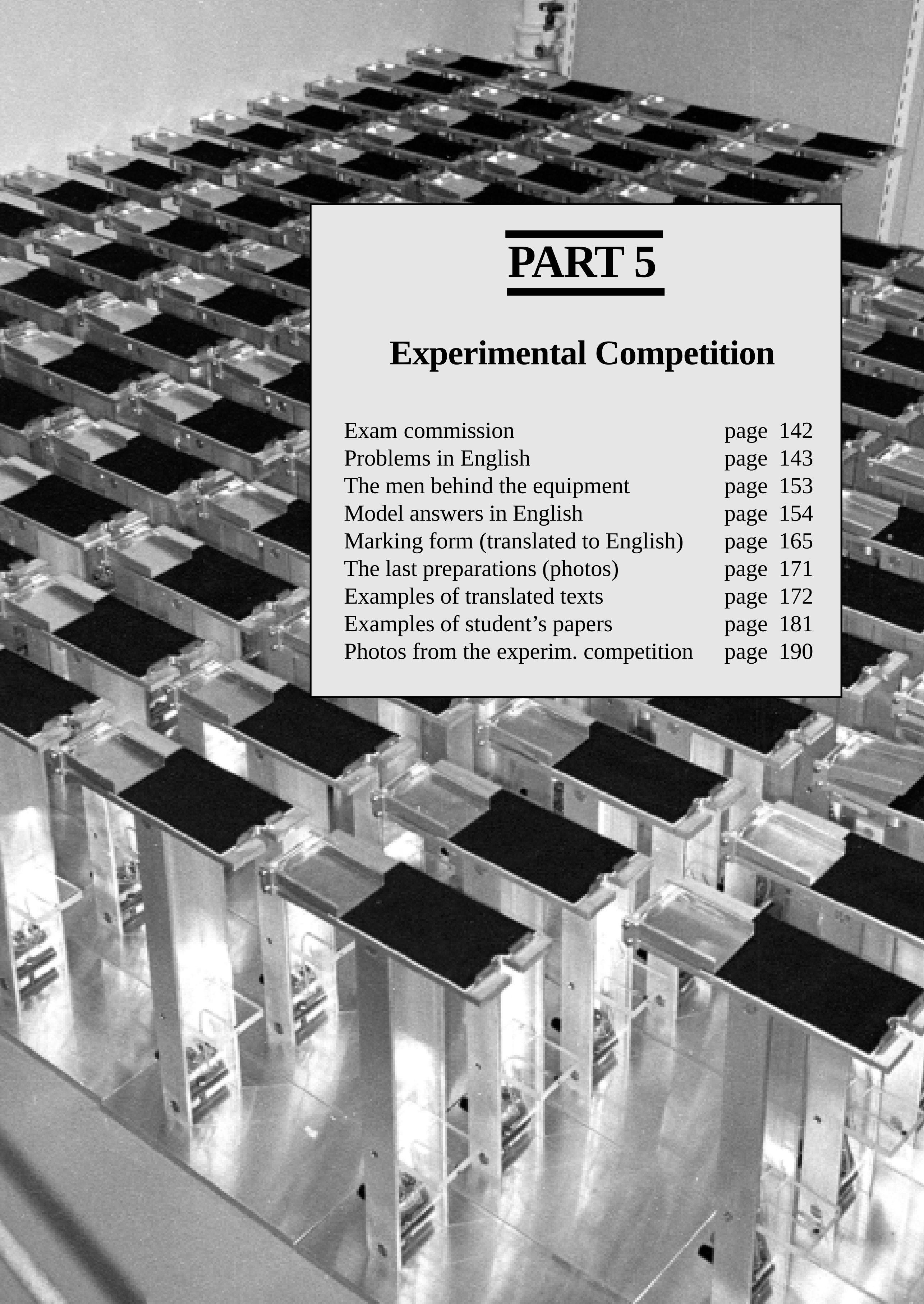

PART 5

Experimental Competition

Preparation of the experimental competition was carried out by:

Børge Holme

Tom Henning Johansen

Arnt Inge Vistnes

Photo: Geir Holm

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27th INTERNATIONAL PHYSICS OLYMPIAD
OSLO, NORWAY

EXPERIMENTAL COMPETITION
JULY 4 1996

Time available: 5 hours

READ THIS FIRST :

1. Use only the pen provided.
2. Use only the marked side of the paper.
3. No points will be given for error estimates except in 2c. However, it is expected that the correct number of significant figures are given.
4. When answering problems, use as little text as possible. You get full credit for an answer in the form of a numerical value, a drawing, or a graph with the proper definition of axes, etc.
5. Write on top of *every* sheet in your report:
   - Your candidate number (IPhO ID number)
   - The section number
   - The number of the sheet
6. Write on the front page the total number of sheets in your report, including graphs, drawings etc.
7. Ensure to include in your report the last page in this set used for answering section 2a and 3b, as well as all graphs requested.

SAFETY HAZARD: Be careful with the two vertical blades on the large stand. The blades are sharp!

This set of problems consists of 10 pages.

SUMMARY

The set of problems will cover a number of topics in physics. First, some mechanical properties of a physical pendulum will be explored, and you should be able to determine the acceleration of gravity. Then, magnetic forces are added to the pendulum. In this part the magnetic field from a permanent magnet is measured using an electronic sensor. The magnetic moment of a small permanent magnet will be determined. In addition, a question in optics in relation to the experimental setup will be asked.

INSTRUMENTATION

The following equipment is available (see Figure 1):

A	Large aluminium stand
B	Threaded brass rod with a tiny magnet in one end (painted white) (iron in the other).
C	2 Nuts with a reflecting surface on one side
D	Oscillation period timer (clock) with digital display
E	Magnetic field (Hall) probe, attached to the large stand
F	9 V battery
G	Multimeter, Fluke model 75
H	2 Leads
I	Battery connector
J	Cylindrical stand made of PVC (grey plastic material)
K	Threaded rod with a piece of PVC and a magnet on the top
L	Small PVC cylinder of length 25.0 mm (to be used as a spacer)
M	Ruler

If you find that the large stand wiggles, try to move it to a different posistion on your table, or use a piece of paper to compensate for the non-flat surface.

The **pendulum** should be mounted as illustrated in Figure 1. The long threaded rod serves as a physical pendulum, hanging in the large stand by one of the nuts. The groove in the nut should rest on the two vertical blades on the large stand, thus forming a horizontal axis of rotation. The reflecting side of the nut is used in the oscillation period measurement, and should always face toward the timer.

The **timer** displays the period of the pendulum in seconds with an uncertainty of ±1 ms. The timer has a small infrared light source on the right-hand side of the display (when viewed from the front), and an infrared detector mounted close to the emitter. Infrared light from the emitter is reflected by the mirror side of the nut. The decimal point lights up when the reflected light hits the detector. For proper detection the timer can be adjusted vertically by a screw (see N in Figure 1). Depending on the adjustment, the decimal point will blink either once or twice each oscillation period. When it blinks twice, the display shows the period of oscillation, T. When it blinks once, the displayed number is $2T$. Another red dot appearing after the *last* digit indicates low battery. If battery needs to be replaced, ask for assistance.

The **multimeter** should be used as follows:
Use the “VΩ” and the “COM” inlets. Turn the switch to the DC voltage setting. The display then shows the DC voltage in volts. The uncertainty in the instrument for this setting is ±(0.4%+1 digit).

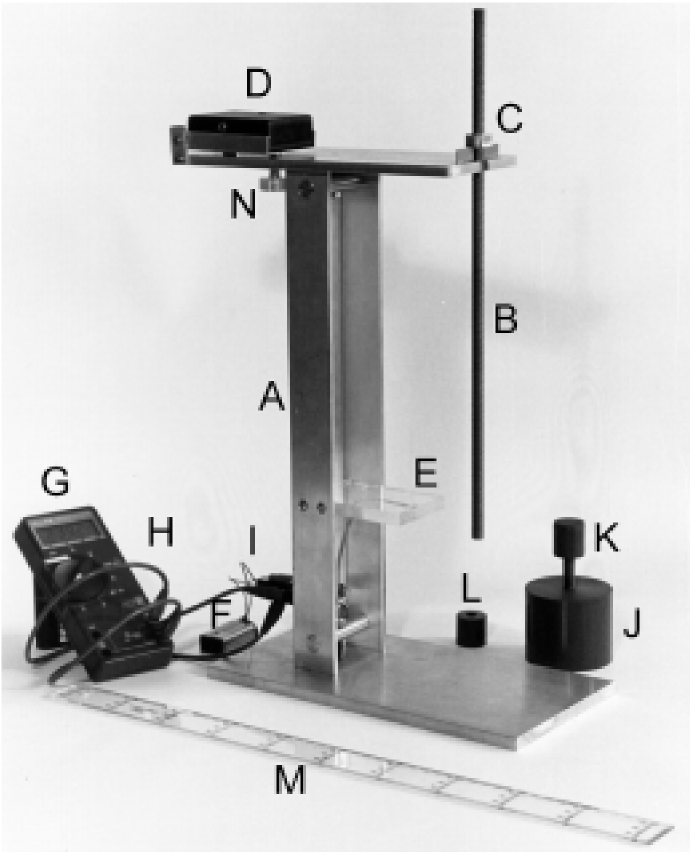


Photo: Geir Holm

Figure 1. The instrumentation used.

SAFETY HAZARD: Be careful with the two vertical blades on the large stand. The blades are sharp!

THE PHYSICAL PENDULUM

A *physical pendulum* is an extended physical object of arbitrary shape that can rotate about a fixed axis. For a physical pendulum of mass M oscillating about a horizontal axis a distance, l, from the centre of mass, the period, T, for small angle oscillations is

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{I}{M\,l} + l} \qquad (1)$$

Here g is the acceleration of gravity, and I is the moment of inertia of the pendulum about an axis parallel to the rotation axis but through the centre of mass.

Figure 2 shows a schematic drawing of the physical pendulum you will be using. The pendulum consists of a cylindrical metal rod, actually a long screw, having length L, average radius R, and at least one nut. The values of various dimensions and masses are summarised in Table 1. By turning the nut you can place it at any position along the rod. Figure 2 defines two distances, x and l, that describe the position of the rotation axis relative to the end of the rod and the centre of mass, respectively.

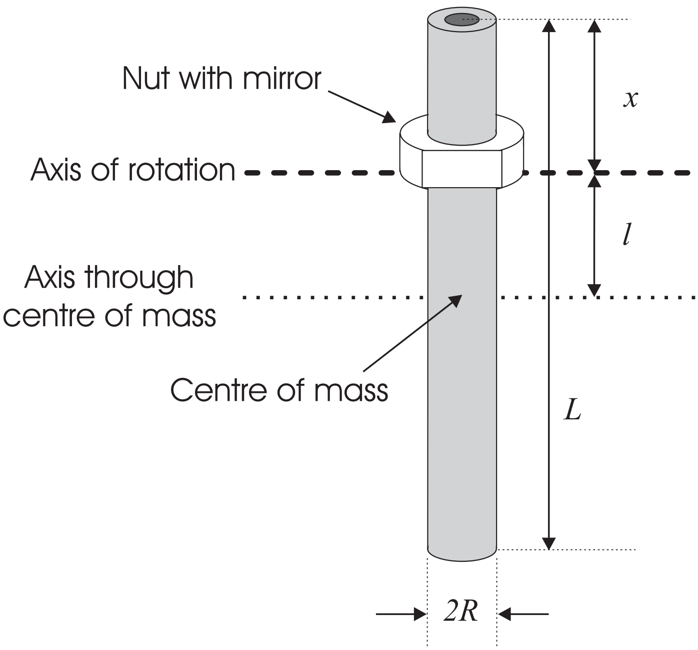


Figure 2: Schematic drawing of the pendulum with definition of important quantities.

Rod

Length	L	(400.0 ± 0.4) mm
Average radius	R	(4.4 ± 0.1) mm
Mass	M_{ROD}	$(210.2 \pm 0.2) \cdot 10^{-3}$ kg
Distance between screw threads		(1.5000 ± 0.0008) mm

Nut

Height	h	(9.50 ± 0.05) mm
Depth of groove	d	(0.55 ± 0.05) mm
Mass	M_{NUT}	$(4.89 \pm 0.03) \cdot 10^{-3}$ kg

Table 1: Dimensions and weights of the pendulum

A reminder from the front page: No points will be given for error estimates except in 2c. However, it is expected that the correct number of significant figures are given.

Section 1 : Period of oscillation versus rotation axis position (4 marks)

a) Measure the oscillation period, T, as a function of the position x, and present the results in a table.

b) Plot T as a function of x in a graph. Let 1 mm in the graph correspond to 1 mm in x and 1 ms in T. How many positions give an oscillation period equal to $T = 950$ ms, $T = 1000$ ms and $T = 1100$ ms, respectively?

c) Determine the x and l value that correspond to the minimum value in T.

Section 2 : Determination of g (5 marks)

For a physical pendulum with a *fixed* moment of inertia, I, a given period, T, may in some cases be obtained for two different positions of the rotation axis. Let the corresponding distances between the rotation axis and the centre of mass be l_1 and l_2. Then the following equation is valid:

$$l_1 l_2 = \frac{I}{M} \qquad (2)$$

a) Figure 6 on the last page in this set illustrates a physical pendulum with an axis of rotation displaced a distance l_1 from the centre of mass. Use the information given in the figure caption to indicate *all* positions where a rotation axis parallel to the drawn axis can be placed without changing the oscillation period.

b) Obtain the local Oslo value for the acceleration of gravity g as accurately as possible. *Hint: There are more than one way of doing this. New measurements might be necessary.* Indicate *clearly* by equations, drawings, calculations etc. the method you used.

c) Estimate the uncertainty in your measurements and give the value of g with error margins.

Section 3 : Geometry of the optical timer (3 marks)

a) Use direct observation and reasoning to characterise, qualitatively as well as quantitatively, the shape of the reflecting surface of the nut (the mirror). (You may use the light from the light bulb in front of you).

Options (several may apply):

1. Plane mirror
2. Spherical mirror
3. Cylindrical mirror
4. Cocave mirror
5. Convex mirror

In case of 2-5: Determine the radius of curvature.

b) Consider the light source to be a point source, and the detector a simple photoelectric device. Make an illustration of how the light from the emitter is reflected by the mirror on the nut in the experimantal setup (side view and top view). Figure 7 on the last page in this set shows a vertical plane through the timer display (front view). Indicate in this figure the whole region where the reflected light hits this plane when the pendulum is vertical.

Section 4 : Measurement of magnetic field (4 marks)

You will now use an electronic sensor (Hall-effect sensor) to measure magnetic field. The device gives a voltage which depends linearly on the vertical field through the sensor. The field-voltage coefficient is $\Delta V / \Delta B = 22.6$ V/T (Volt/Tesla). As a consequence of its design the sensor gives a non-zero voltage (zero-offset voltage) in zero magnetic field. Neglect the earth's magnetic field.

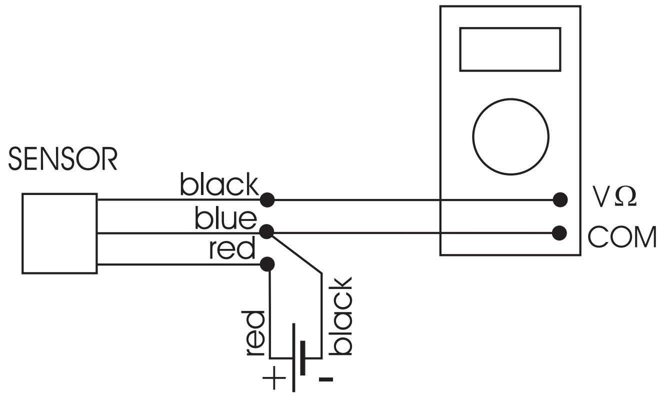


Figure 3: Schematics of the magnetic field detector system

a) Connect the sensor to the battery and voltmeter as shown above. Measure the zero-offset voltage, V_0 .

A permanent magnet shaped as a circular disk is mounted on a separate stand. The permanent magnet can be displaced vertically by rotating the mount screw, which is threaded identically to the pendulum rod. The dimensions of the permanent magnet are; thickness t = 2.7 mm, radius r = 12.5 mm.

b) Use the Hall sensor to measure the vertical magnetic field, B, from the permanent magnet along the cylinder axis, see Figure 4. Let the measurements cover the distance from y = 26 mm (use the spacer) to y = 3.5 mm, *where* y = 1 mm *corresponds to the sensor and permanent magnet being in direct contact*. Make a graph of your data for B versus y.

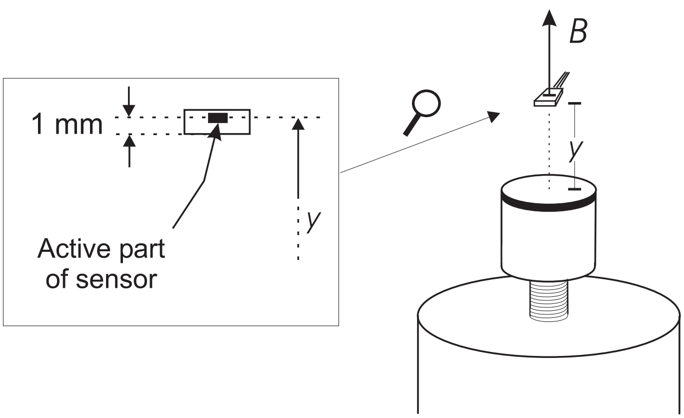


Figure 4: Definition of the distance y between top of magnet and the active part of the sensor.

c) It can be shown that the field along the axis of a cylindrical magnet is given by the formula

$$B(y) = B_0\left[\frac{y+t}{\sqrt{(y+t)^2+r^2}} - \frac{y}{\sqrt{y^2+r^2}}\right] \qquad (3)$$

where t is the cylinder thickness and r is the radius. The parameter B_0 characterizes the strength of the magnet. Find the value of B_0 for your permanent magnet.§ Base your determination on two measured B-values obtained at different y.

Section 5 : Determination of magnetic dipole moment (4 marks)

A tiny magnet is attached to the white end of the pendulum rod. Mount the pendulum on the stand with its magnetic end down and with ***x* = 100 mm**. Place the permanent magnet mount under the pendulum so that both the permanent magnet and the pendulum have common cylinder axis. The alignment should be done with the permanent magnet in its lowest position in the mount. (Always avoid close contact between the permanent magnet and the magnetic end of the pendulum.)

a) Let z denote the air gap spacing between the permanent magnet and the lower end of the pendulum. Measure the oscillation period, T, as function of the distance, z. The measurement series should cover the interval from z = 25 mm to z = 5.5 mm while you use as small oscillation amplitude as possible. Be aware of the possibility that the period timer might display $2T$ (see remark regarding the timer under *Instrumentation* above). Plot the observed T versus z.

b) With the additional magnetic interaction the pendulum has a period of oscillation, T, which varies with z according to the relation

$$\frac{1}{T^2} \propto 1 + \frac{\mu B_0}{Mgl} f(z) \qquad (4)$$

Here $\propto$ stand for "proportional to", and μ is the magnetic dipole moment of the tiny magnet attached to the pendulum, and   is the parameter determined in section 4c. The function *f(z)* includes the variation in magnetic field with distance. In Figure 5 on the next page you find the particular *f(z)* for our experiment, presented as a graph.
Select an appropirate point on the graph to determine the unknown magnetic moment μ.

---

§ $2B_0$ is a material property called remanent magnetic induction, B_r.

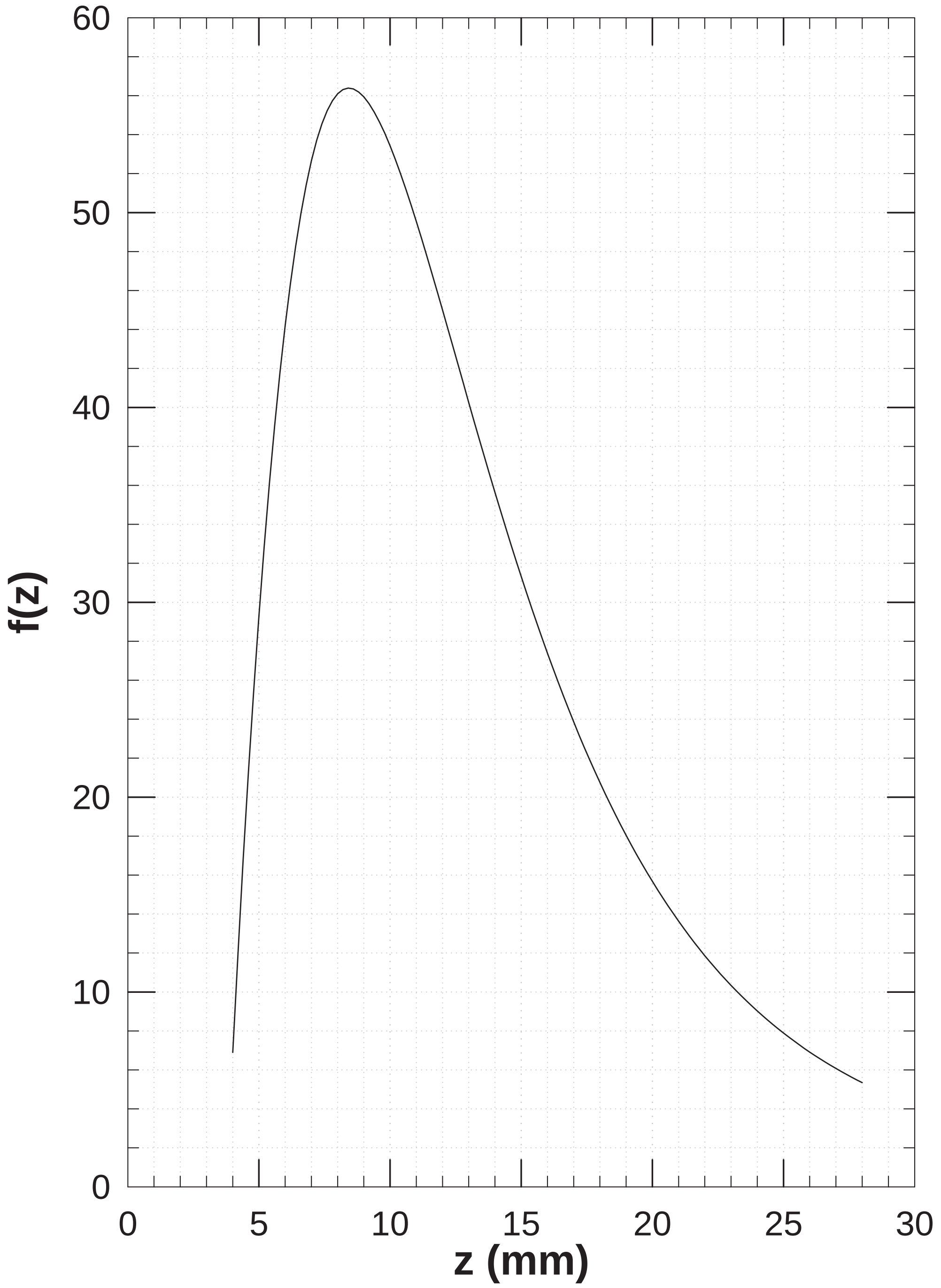


Figure 5. Graph of the dimension-less function f(z) used in section 5b.

Candidate:	Question:	Page ..... of .....

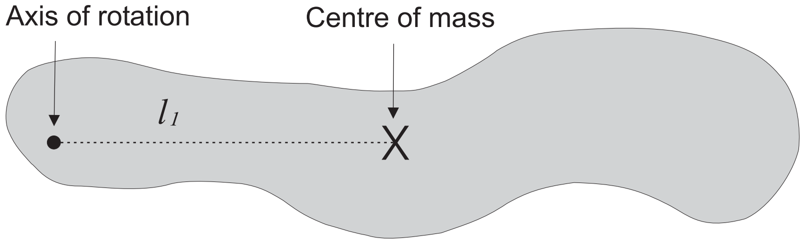


Figure 6. **For use in section 2a**. *Mark* all *positions where a rotation axis (orthogonal to the plane of the paper) can be placed without changing the oscillation period. Assume for this pendulum (drawn on scale, 1:1) that $I/M = 2100\ mm^2$*. (Note: In this booklet the size of this figure is about 75% of the size in the original examination paper.)

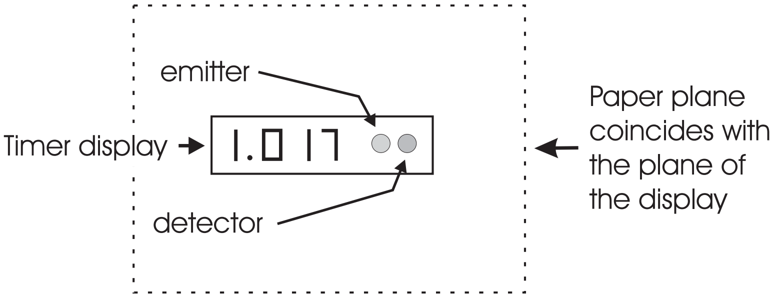


Figure 7. **For use in section 3b.** *Indicate the whole area where the reflected light hits when the pendulum is vertical.*

Include this page in your report!

The men behind the equipment

The equipment for the practical competition was constructed and manufactured at the Mechanics Workshop at the Department of Physics, University of Oslo (see picture below, from left to right: Tor Enger (head of the Mechanics Workshop), Pål Sundbye, Helge Michaelsen, Steinar Skaug Nilsen, and Arvid Andreassen).

Photo: Geir Holm

The electronic timer was designed and manufactured by Efim Brondz, Department of Physics, University of Oslo (see picture below). About 40.000 soldering points were completed manually, enabling the time-recording during the exam to be smooth and accurate.

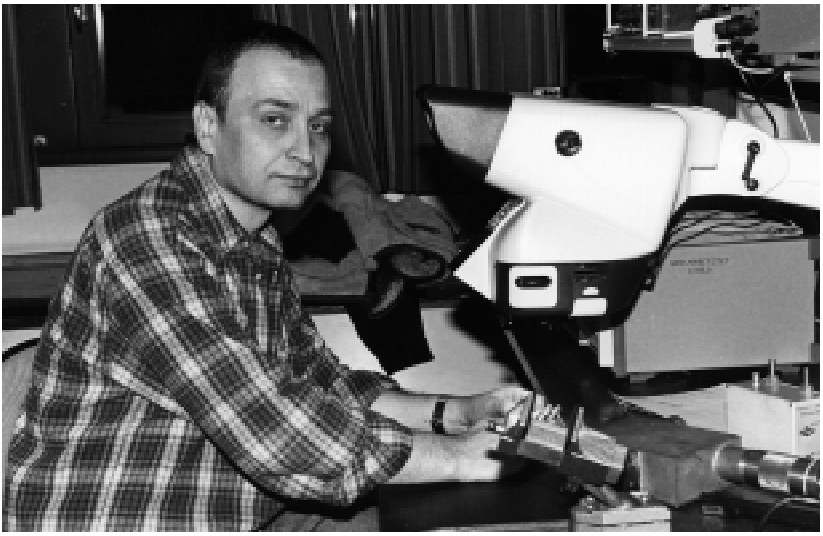


Photo: Geir Holm

27th INTERNATIONAL PHYSICS OLYMPIAD OSLO, NORWAY

Model Answer

for the

EXPERIMENTAL COMPETITION

JULY 4 1996

These model answers indicate what is required from the candidates to get the maximum score of 20 marks. Some times we have used slightly more text than required; paragraphs written in italic give additional comments. This practical exam will reward students with creativity, intuition and a thorough understanding of the physics involved.

> *Alternative solutions regarded as less elegant or more time consuming are printed in frames like this with white background.*

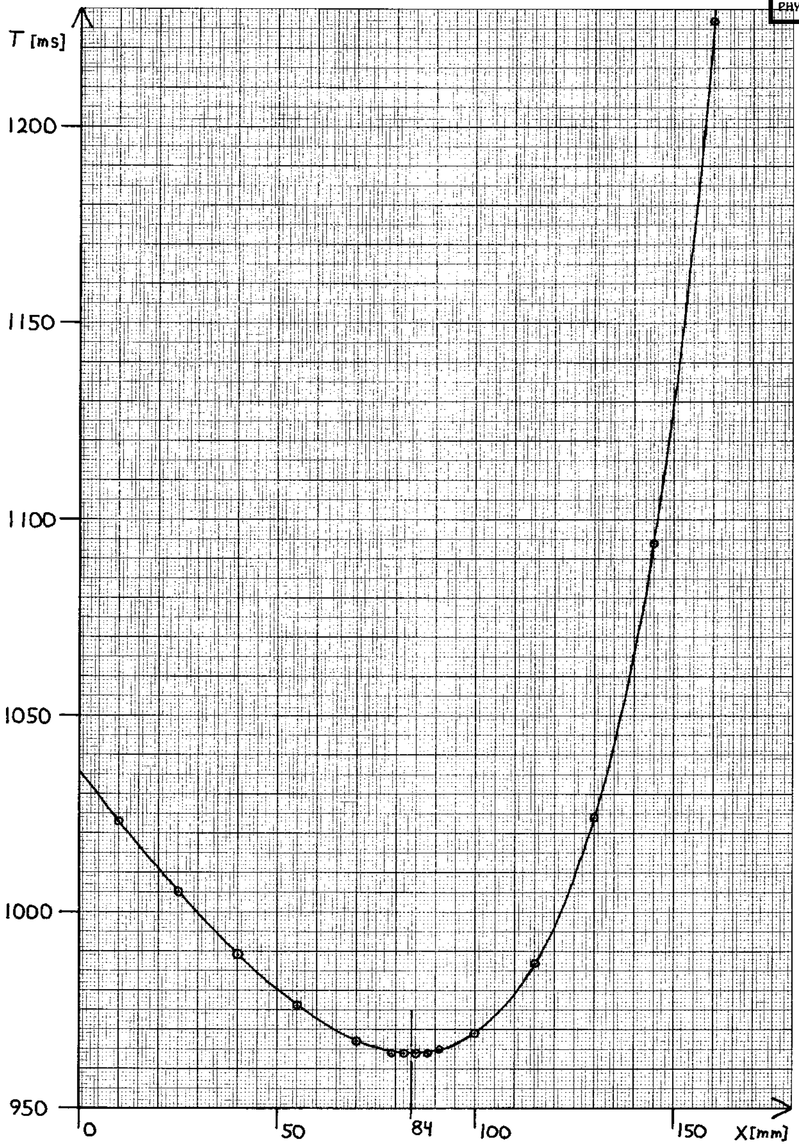
> *Anticipated INCORRECT answers are printed on grey background and are included to point out places where the students may make mistakes or approximations without being aware of them.*

Section 1:

1a) Threads are 1.50 mm/turn. Counted turns to measure position *x*.

Turn no.	0	10	20	30	40	50	60	70	80	90	100
x [mm]	10.0	25.0	40.0	55.0	70.0	85.0	100.0	115.0	130.0	145.0	160.0
T [ms]	1023	1005	989	976	967	964	969	987	1024	1094	1227

Turn no.	110	120	46	48	52	54
x [mm]	175.0	190.0	79.0	82.0	88.0	91.0
T [ms]	1490	2303	964	964	964	965



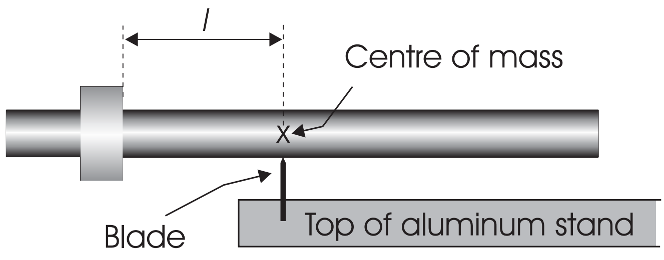
1b) Graph: *T(x)*, shown above.

T = 950 ms: NO positions
T =1000 ms: 2 positions
T =1100 ms: 1 position

If the answer is given as corresponding x-values, and these reflect the number of positions asked for, this answer will also be accepted.

1c) Minimum on graph: x = 84 mm, (estimated uncertainty 1 mm)

By balancing the pendulum horizontally: l = 112.3 mm + 0.55 mm = 113 mm



ALTERNATIVE 1c-1:

$$x_{CM} = \frac{M_{ROD}L - M_{NUT}h}{2M} + \frac{M_{NUT}}{M}x = 197.3 \text{ mm for } x = 84 \text{ mm}$$

gives l = 197.3 mm - 84 mm = 113 mm

$M = M_{ROD} + M_{NUT}$, h = 8.40 mm = height of nut minus two grooves.

INCORRECT 1c-1: Assuming that the centre of mass for the pendulum coincides with the midpoint, L/2, of the rod gives $l = L/2 - x$ = 116 mm.

(The exact position of the minimum on the graph is x = 84.4 mm. with l = 112.8 mm)

Section 2:

2a) $$l_2 = \frac{I}{Ml_1} = \frac{2100 \text{ mm}^2}{60 \text{ mm}} = 35 \text{ mm}$$

See also Figure 6 on the next page

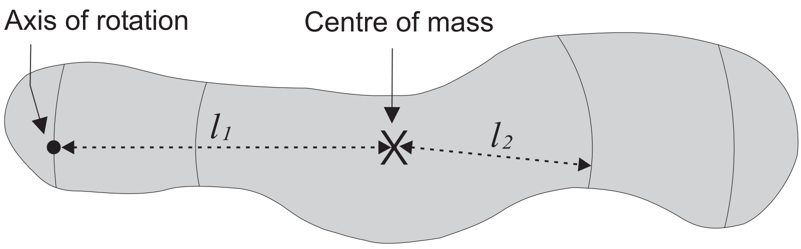


Figure 6. **For use in section 2a**. *Mark* all *positions where a rotation axis (orthogonal to the plane of the paper) can be placed without changing the oscillation period. Assume for this pendulum (drawn on scale, 1:1) that $I/M = 2100\ mm^2$.* (Note: In this booklet the size of this figure is about 75% of the size in the original examination paper.)

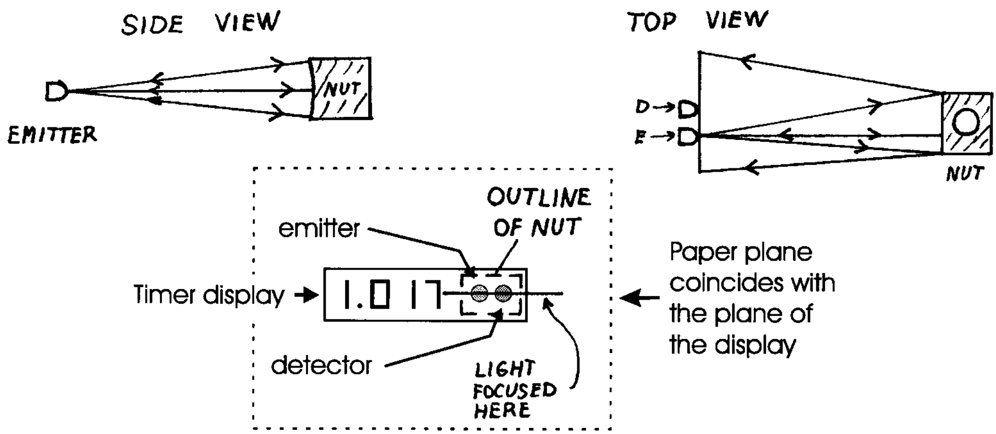


Figure 7. **For use in section 3b.** *Indicate the whole area where the reflected light hits when the pendulum is vertical.*

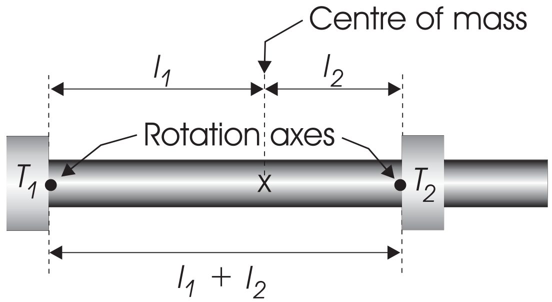
Include this page in your report!

2b) Simple method with small uncertainty: Inverted pendulum.

Equation (1) + (2) $\Rightarrow T_1 = T_2 = \frac{2\pi}{\sqrt{g}}\sqrt{l_1 + l_2} \Leftrightarrow g = \frac{4\pi^2}{T_1^2}(l_1 + l_2)$

NOTE: Independent of I/M !

Used both nuts with one nut at the end to maximise $l_1 + l_2$. Alternately adjusted nut positions until equal periods $T_1 = T_2$:



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$T_1 = T_2$= 1024 ms.
Adding the depth of the two grooves to the measured distance between nuts:
$l_1 + l_2$ = (259.6 + 2 · 0.55) mm = 0.2607 m

$$g = \frac{4\pi^2}{T_1^2}(l_1 + l_2) = \frac{4 \cdot 3.1416^2 \cdot 0.2607\text{m}}{(1.024\text{s})^2} = \underline{\underline{9.815\text{m}/\text{s}^2}}$$

ALTERNATIVE 2b-1: Finding I(x). *Correct but time consuming.*
It is possible to derive an expression for I as a function of x. By making sensible approximations, this gives:

$$\frac{I(x)}{M} = \left[\frac{L^2}{12} + \frac{M_{NUT}}{M}\left(\frac{L+h}{2} - x\right)^2\right]\frac{M_{ROD}}{M}$$

which is accurate to within 0.03 %. Using the correct expression for l as a function of x:

$$l(x) = x_{CM} - x = \frac{M_{ROD}L - M_{NUT}h}{2M} - \frac{M_{ROD}}{M}x \ = 195.3 \text{ mm} - 0.9773x,$$

equation (1) can be used on any point (x, T) to find g. Choosing the point (85 mm, 964 ms) *gives:*

$$g = \frac{4\pi^2}{T^2}\left[\frac{I(x)}{M \cdot l(x)} + l(x)\right] = \frac{4 \cdot 3.1416^2 \cdot 0.2311\text{m}}{(0.964\text{s})^2} = \underline{\underline{9.818 \text{ m}/\text{s}^2}}$$

Using the minimum point on the graph in the way shown below is wrong, since the curve in ***1b)*** *,* $T(x) = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{I(x)}{M \cdot l(x)} + l(x)}$ *with I(x)/M and l(x) given above, describes a continuum of* ***different*** *pendulums with changing I(x) and moving centre of mass.*
Equation (1): $T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{I}{Ml} + l}$ *describes* ***one*** *pendulum with fixed I, and does not apply to the curve in* ***1b).***

INCORRECT 2b-1: At the minimum point we have from Equation (2) and **1c)**:

$l_1 = l_2 = l = \sqrt{I/M} = (113 \pm 1)$ mm Equation (1) becomes

$$T_{\min} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{l^2}{l} + l} = \frac{2\pi}{\sqrt{g}}\sqrt{2l}$$ and

$$g = \frac{8\pi^2 l}{T_{\min}{}^2} = \frac{8 \cdot 3.1416^2 \cdot 0.113\,\mathrm{m}}{(0.964\,\mathrm{s})^2} = 9.60\ \mathrm{m/s^2}$$

Another source of error which may accidentally give a reasonable value is using the wrong value l = (116 ± 1)mm from «INCORRECT 1c-1»:

INCORRECT 2b-2: $g = \frac{8\pi^2 l}{T_{\min}{}^2} = \frac{8 \cdot 3.1416^2 \cdot 0.116\,\mathrm{m}}{(0.964\,\mathrm{s})^2} = 9.86\ \mathrm{m/s^2}$

Totally neglecting the mass of the nut but remembering the expression for the moment of inertia for a thin rod about a perpendicular axis through the centre of mass, $I = ML^2/12$*, gives from equation (2) for the minimum point:* $l^2 = I/M = L^2/12 = 0.01333\ m^2$*. This value is accidentally only 0.15% smaller than the correct value for I(x)/M at the minimum point on the curve in* ***1b)****:*

$$\frac{I(x = 84.43\,\mathrm{mm})}{M} = \left[\frac{L^2}{12} + \frac{M_{NUT}}{M}\left(\frac{L+h}{2} - x\right)^2\right]\frac{M_{ROD}}{M} = 0.01335\ \mathrm{m}^2.$$

(continued on next page)

(cont.)

Neglecting the term $\frac{M_{NUT}}{M}\left(\frac{L+h}{2}-84.43\,\text{mm}\right)^2 = 0.00033\ \text{m}^2$ *is nearly compensated by omitting the factor* $\frac{M_{ROD}}{M}$ =0.977. *However, each of these approximations are of the order of 2.5 %, well above the accuracy that can be achieved.*

INCORRECT 2b-3: At the minimum point equation (2) gives $l^2 = \frac{I}{M} = \frac{L^2}{12}$. Then

$$T_{\min} = \frac{2\pi}{\sqrt{g}}\sqrt{2l} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{2L}{\sqrt{12}}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{L}{\sqrt{3}}} \quad \text{and}$$

$$g = \frac{4\pi^2 L}{\sqrt{3}\,T_{\min}{}^2} = \frac{4\cdot 3.1416^2\cdot 0.4000\ \text{m}}{1.7321\cdot(0.964\ \text{s})^2} = 9.81\ \text{m/s}^2$$

2c) Estimating uncertainty in the logarithmic expression for g:

Let $S \equiv l_1 + l_2 \quad \Rightarrow \quad g = \frac{4\pi^2 S}{T^2}$

$\Delta S = 0.3$ mm $\qquad \Delta T = 1$ ms

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(-2\frac{\Delta T}{T}\right)^2} = \sqrt{\left(\frac{0.3\ \text{mm}}{260.7\ \text{mm}}\right)^2 + \left(2\cdot\frac{1\ \text{ms}}{1024\ \text{ms}}\right)^2}$$

$$= \sqrt{(0.0012)^2 + (0.0020)^2} = 0.0023 = 0.23\%$$

$$\Delta g = 0.0023\cdot 9.815\ \text{m/s}^2 = 0.022\ \text{m/s}^2$$

$$\underline{\underline{g = (9.82 \pm 0.02)\ \text{m/s}^2}}$$

The incorrect methods INCORRECT 2b-1, 2b-2 and 2b-3 have a similar expressions for g as above. With Δl = 1 mm in INCORRECT 2b-1 and 2b-2 we get Δg = 0.09 m/s^2.
INCORRECT 2b-3 should have Δl = 0.3 mm and Δg = 0.02 m/s^2.

ALTERNATIVE 3 has a very complicated x dependence in g. Instead of differentiating g(x) it is easier to insert the two values x+Δx and x-Δx in the expression in brackets [ ], thus finding an estimate for Δ[ ] and then using the same formula as above.

(The official local value for g, measured in the basement of the adjacent building to where the practical exam was held is g = 9.8190178 m/s^2 with uncertainty in the last digit.)

Section 3.

3a) 3. Cylindrical mirror
4. Concave mirror

Radius of curvature of cylinder, r = 145 mm. *(Uncertainty approx. ± 5 mm, not asked for.)*

(In this set-up the emitter and detector are placed at the cylinder axis. The radius of curvature is then the distance between the emitter/detector and the mirror.)

3b) Three drawings, see Figure 7 on page 4 in this Model Answers.

(The key to understanding this set-up is that for a concave cylindrical mirror with a point source at the cylinder axis, the reflected light will be focused back onto the cylinder axis as a line segment of length twice the width of the mirror.)

Section 4.

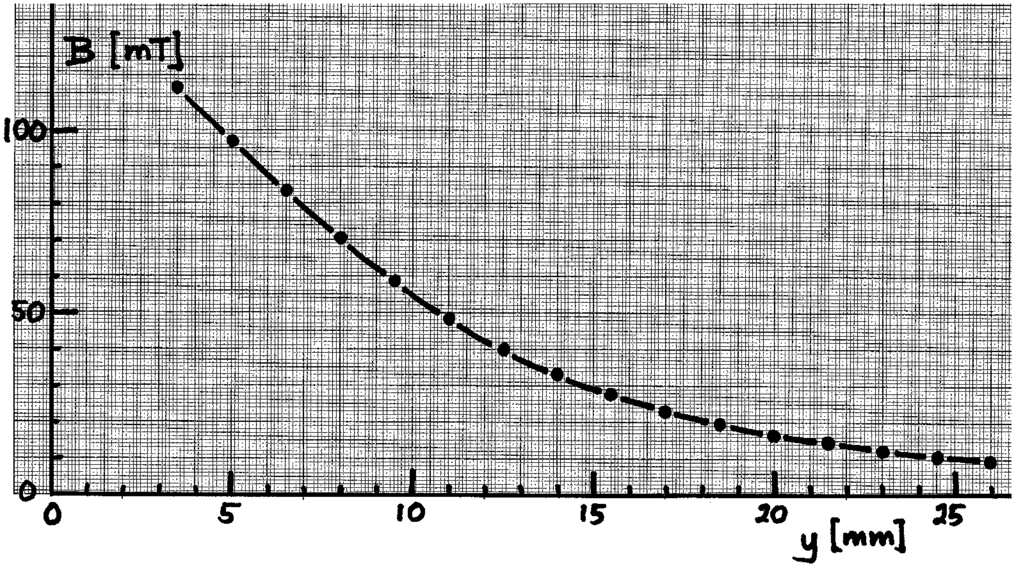
4a) V_o = 2.464 V *(This value may be different for each set-up.)*

4b) Threads are 1.50 mm /turn. Measured *V(y)* for each turn. Calculated

$$B(y) = [V(y) - V_0]\frac{\Delta B}{\Delta V} = [V(y) - V_0] \,/\, \frac{\Delta V}{\Delta B}.$$ *(Table not requested)*

See graph on next page.

Graph: B(y):



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4c)

$$B_0 = B(y)\left[\frac{y+t}{\sqrt{(y+t)^2+r^2}} - \frac{y}{\sqrt{y^2+r^2}}\right]^{-1}$$

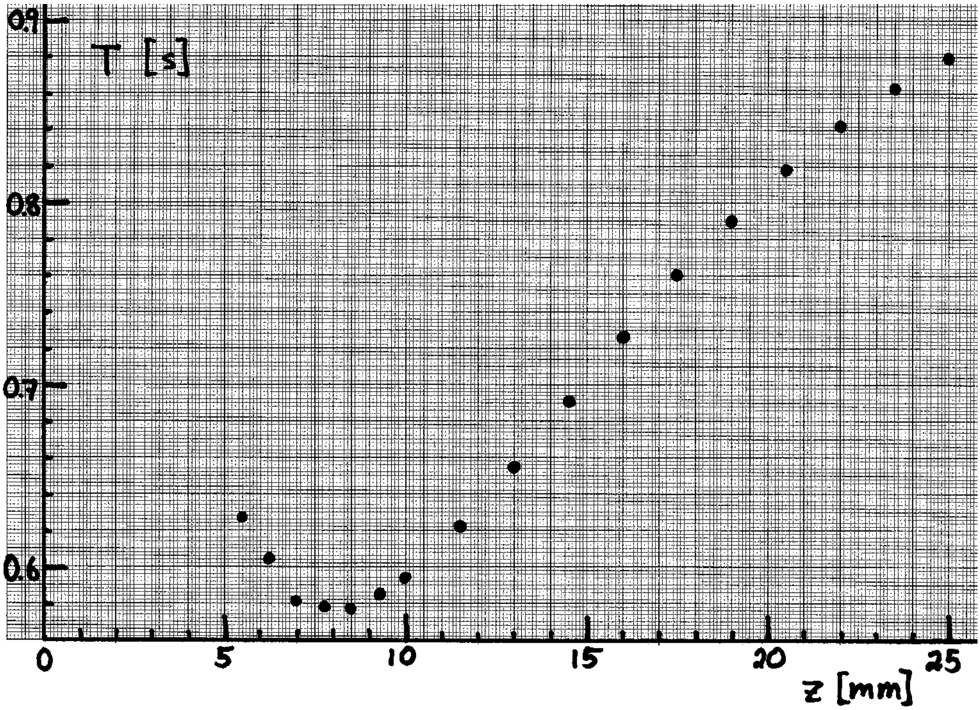
The point (11 mm, 48.5 mT) gives B_0 = 0.621 T and (20 mm, 16,8 mT) gives B_0 = 0.601 T.
Mean value: B_0 = 0.61 T *(This value may vary for different magnets.)*

Section 5:

5a) Used the spacer and measured *T(z)* from z = 25 mm to 5.5 mm. *(Table is not requested.)*

See plot on next page.

Graph: T(z):



5b) $l(x = 100 \text{ mm}) = 97.6$ mm *(by balancing the pendulum or by calculation as in* ***1c)****.*

$$M = M_{ROD} + M_{NUT}$$

Proportionality means: $\dfrac{1}{T^2} = a\left[1 + \dfrac{\mu B_0}{Mgl} f(z)\right]$ where a is a proportionality constant. Setting $B_0 = 0$ corresponds to having an infinitely weak magnet or no magnet at all. Removing the large magnet gives: $T_0 = 968$ ms and $\dfrac{1}{T_0{}^2} = a\left[1 + 0 \cdot \dfrac{\mu}{Mgl} f(z)\right]$ or $a = \dfrac{1}{T_0{}^2}$.

Selecting the point where *f(z)*, see Fig. 5, changes the least with *z*, i.e., at the maximum, one has $f_{max} = 56.3$. This point must correspond to the minimum oscillation period, which is measured to be $T_{min} = 576$ ms.

We will often need the factor

$$\frac{Mgl}{B_0} = \frac{0.215 \text{ kg} \cdot 9.82 \text{ m/s}^2 \cdot 0.0976 \text{ m}}{0.61 \text{ T}} = 0.338 \text{ Am}^2.$$

.

The magnetic moment then becomes

$$\mu = \frac{Mgl}{B_0}\frac{1}{f_{max}}\left[\left(\frac{T_0}{T}\right)^2 - 1\right] = \frac{0.338 \text{ Am}^2}{56.3}\left[\left(\frac{968}{576}\right)^2 - 1\right] = \underline{\underline{1.1 \cdot 10^{-2} \text{ Am}^2}}$$

ALTERNATIVE 5b-1: *Not what is asked for:* Using *two* points to eliminate the proportionality constant *a*: Equation (4) or $\frac{1}{T^2} = a\left[1 + \frac{\mu B_0}{Mgl} f(z)\right]$ gives:

$$aT_1^2\left[1 + \frac{\mu B_0}{Mgl} f(z_1)\right] = aT_2^2\left[1 + \frac{\mu B_0}{Mgl} f(z_2)\right]$$

$$T_1^2 + T_1^2 \frac{\mu B_0}{Mgl} f(z_1) = T_2^2 + T_2^2 \frac{\mu B_0}{Mgl} f(z_2)$$

$$\frac{\mu B_0}{Mgl}\left[T_1^2 f(z_1) - T_2^2 f(z_2)\right] = T_2^2 - T_1^2$$

$$\mu = \frac{Mgl}{B_0} \cdot \frac{T_2^2 - T_1^2}{T_1^2 f(z_1) - T_2^2 f(z_2)}$$

Choosing two points (z_1 = 7 mm, T_1 = 580.5 ms) and (z_2 = 22 mm, T_2 = 841ms). Reading from the graph $f(z_1)$ = 56.0 and $f(z_2)$ = 12.0 we get

$$\mu = 0.338 \text{ Am}^2 \cdot \frac{841^2 - 580^2}{580^2 \cdot 56.0 - 841^2 \cdot 12.0} = \underline{1.2 \cdot 10^{-2} \text{ Am}^2}$$

Candidate:	**Total score:** + + + + =
Country:	Marker's name:
Language:	Comment:

Marking Form

for the *Experimental Competition* at the 27th International Physics Olympiad Oslo, Norway

July 4, 1996

To the marker: *Carefully read through the candidate's exam papers and compare with the model answer. You may use the transparencies (handed out) when checking the graph in* ***1b)*** *and the drawing in* ***2a)****. When encountering words or sentences that require translation, postpone marking of this part until you have consulted the interpreter.*

Use the table below and mark a circle around the point values to be subtracted. Add vertically the points for each subsection and calculate the score.
NB: Give score 0 if the value comes out negative for any subsection.
Add the scores for each subsection and write the sum in the 'Total score'- box at the upper right. Keep decimals all the way.

If you have questions, consult the marking leader. Good luck, and remember that you will have to defend your marking in front of the team leaders.

(Note: The terms "INCORRECT 2b-1" found in the table for subsection 2c) and similar terms elsewhere, refer to the Model Answer, in which anticipated incorrect answers were included and numbered for easy reference.)

Subsection 1a)	*Deficiency*	*Subtract*
	No answer	**1.0**
	x lacks unit	0.1
	Other than 0 or 1 decimal in x	0.1
	x does not cover the interval 10 mm - 160 mm	0.1
	T lacks unit	0.1
	T given with other than 1 or 0.5 millisecond accuracy	0.1
	Fewer than 11 measuring points (15 mm sep.). Subtr. up to	0.2
	Systematic error in x (e.g. if measured from the top of the nut so that the first x = 0 mm)	0.2
	If not aware of doubling of the timer period	0.2
Other (specify):		
	Score for subsection 1a: **1.0 -**	=

Subsection 1b)	*Deficiency*	*Subtract*
	No answer	**1.0**
	Lacks "x [(m)m]" on horizontal axis	0.1
	1 mm on paper does not correspond to 1 mm in x	0.1
	Fewer than 3 numbers on horizontal axis	0.1
	Lacks "T [(m)s]" on vertical axis	0.1
	1 mm on paper does not correspond to 1 ms in T	0.1
	Fewer than 3 numbers on vertical axis	0.1
	Measuring points not clearly shown (as circles or crosses)	0.2
	More than 5 ms deviation in more than 2 measuring points on the graph	0.2
	Wrong answer to the questions (x-values give full score if correct number of values: 0, 2, 1)	0.2
Other (specify):		
	Score for subsection 1b): **1.0 -**	=

Subsection 1c)	*Deficiency*	*Subtract*
	No answer	**2.0**
	x outside the interval 81 - 87 mm. Subtract up to	0.4
	x lacks unit	0.1
	x given more (or less) accurately than in whole millimeters	0.3
	l lacks unit	0.1
	l given more (or less) accurately than the nearest mm	0.3
	Wrong formula (e.g. l = 200.0 mm - x) or something other than $l = x_{CM} - x$	0.6
	If it is not possible to see which method was used to find the center of mass	0.2
Other (specify):		
	Score for subsection 1c): **2.0 -**	=

Subsection 2a)	*Deficiency*	*Subtract*
	No answer	**1.5**
	If drawn straight (vertical) lines	0.4
	If **points** are drawn	0.5
	Other than 4 regions are drawn	0.5
	Inaccurate drawing (> ± 2 mm)	0.3
	Lacks the values l_1 = 60 mm, l_2 = 35 mm on figure or text	0.3
Other (specify):		
	Score for subsection 2a): **1.5 -**	=

Subsection 2b)	*Deficiency*	*Subtract*
	No answer	**2.5**
	Lacks (derivation of) formula for g	0.3
	For INVERTED PENDULUM: Lacks figure	0.2
	Values from possible new measurements not given	0.3
	Incomplete calculations	0.3
	If hard to see which method was used	0.4
	Used the formula for INVERTED PENDULUM but read l_1 and l_2 from graph in **1b)** by a horizontal line for a certain T	1.5
	Used one of the other incorrect methods	2.0
	Other than 3 (or 4) significant figures in the answer	0.3
	g lacks unit m/s^2	0.1
Other (specify):		
	Score for subsection 2b): **2.5 -**	=

Subsection 2c)	*Deficiency*	*Subtract*
	No answer	**2.5**
	Wrong expression for $\Delta g/g$ or Δg. Subtract up to	0.5
	For INVERTED PENDULUM: If 0.3 mm > $\Delta(l_1+l_2)$ > 0.5 mm	0.2
	For ALTERNATIVE 2c-1: If $\Delta[]$ > 0.1 mm	0.2
	For INCORRECT 2c-1 and 2c-2: If 1 mm > Δl > 2 mm	0.2
	For INCORRECT 2c-3: If 0.3 mm > ΔL > 0.4 mm	0.2
	For all methods: If $\Delta T \neq$ 1 (or 0.5) ms	0.2
	Error in the calculation of Δg	0.2
	Lacks answer including $g \pm \Delta g$ with 2 decimals	0.2
	$g \pm \Delta g$ lacks unit	0.1
Other (specify):		
	Score for subsection 2c): **2.5 -**	=

Subsection 3a)	*Deficiency*	*Subtract*
	No answer	**1.0**
	Lacks point **3. cylindrical mirror**	0.3
	Lacks point **4. concave mirror**	0.3
	Includes other points (1, 2 or 5), subtract per wrong point:	0.3
	Lacks value for radius of curvature	0.4
	If $r < 130$ mm or $r > 160$ mm, subtract up to	0.2
	If r is given more accurately than hole millimeters	0.2
Other (specify):		
	Score for subsection 3a): **1.0 -**	=

Subsection 3b)	*Deficiency*	*Subtract*
	No answer	**2.0**
	Lacks side view figure	0.6
	Errors or deficiencies in the side view figure. Subtract up to	0.4
	Lacks top view figure	0.6
	Errors or deficiencies in the top view figure. Subtract up to	0.4
	Drawing shows light focused to a **point**	0.3
	Drawing shows light spread out over an ill defined or wrongly shaped surface	0.3
	Line/surface is not horizontal	0.2
	Line/point/surface not centered symmetrically on detector	0.2
	Line/point/surface has length different from twice the width of the nut (i.e. outside the interval 10 - 30 mm)	0.1
Other (specify):		
	Score for subsection 3b): **2.0 -**	=

Subsection 4a)

	Deficiency	*Subtract*
	No answer	**1.0**
	V_o lacks unit V	0.1
	Less than 3 decimals in V_o	0.1
	Incorrect couplings (would give V_o < 2.3 V or V_o > 2.9 V)	0.8
Other (specify):		
	Score for subsection 4a): **1.0 -**	=

Subsection 4b)

	Deficiency	*Subtract*
	No answer	**1.5**
	Forgotten V_o or other errors in formula for B	0.2
	Lacks "*y* [(m)m]" on horizontal axis	0.1
	Fewer than 3 numbers on horizontal axis	0.1
	Lacks "*B* [(m)T]" on vertical axis	0.1
	Fewer than 3 numbers on vertical axis	0.1
	Fewer than 9 measuring points. Subtract up to	0.2
	Measuring points do not cover the interval 3.5 mm - 26 mm	0.2
	Measuring points not clearly shown (as circles or crosses)	0.1
	Error in data or unreasonably large spread in measuring points. Subtract up to	0.5
Other (specify):		
	Score for subsection 4b): **1.5 -**	=

Subsection 4c)

	Deficiency	*Subtract*
	No answer	**1.5**
	Incorrect formula for B_o	0.3
	If used only one measuring point	0.4
	If used untypical points on the graph	0.3
	Errors in calculation of mean value for B_o	0.2
	B_o lacks unit T	0.1
	Other than two significant figures in (the mean value of) B_o	0.2
	B_o < 0.4 T or B_o > 0.7 T. Subtract up to	0.2
Other (specify):		
	Score for subsection 4c): **1.5 -**	=

Subsection 5a)	Deficiency	Subtract
	No answer	**1.0**
	Lacks "z [(m)m]" on horizontal axis	0.1
	Fewer than 3 numbers on horizontal axis	0.1
	Lacks "T [(m)s]" on vertical axis	0.1
	Fewer than 3 numbers on vertical axis	0.1
	Fewer than 8 measuring points. Subtract up to	0.2
	Measuring points not clearly shown (as circles or crosses)	0.1
	Measuring points do not cover the interval 5.5 mm - 25 mm	0.2
	Error in data (e.g. plotted 2T instead of T) or unreasonably large spread in measuring points. Subtr. up to	0.5
Other (specify):		
	Score for subsection 5a): **1.0 -**	=

Subsection 5b)	Deficiency	Subtract
	No answer	**3.0**
	Forgotten center of mass displacement in l (used l = 100 mm)	0.3
	Used ALTERNATIVE 5b-1	1.0
	Lacks method for finding the proportionality factor a	2.5
	Not found correct proportionality factor a	0.3
	Used another point than the maximum of $f(z)$	0.1
	Incorrect reading of $f(z)$	0.1
	Used M_{ROD} or another incorrect value for M	0.2
	Incorrect calculation of μ	0.3
	μ lacks unit (Am^2 or J/T)	0.2
	More than 2 significant figures in μ	0.3
Other (specify):		
	Score for subsection 5b): **3.0 -**	=

Total points:

Total for section 1 (max. 4 points):
Total for section 2 (max. 5 points):
Total for section 3 (max. 3 points):
Total for section 4 (max. 4 points):
Total for section 5 (max. 4 points):

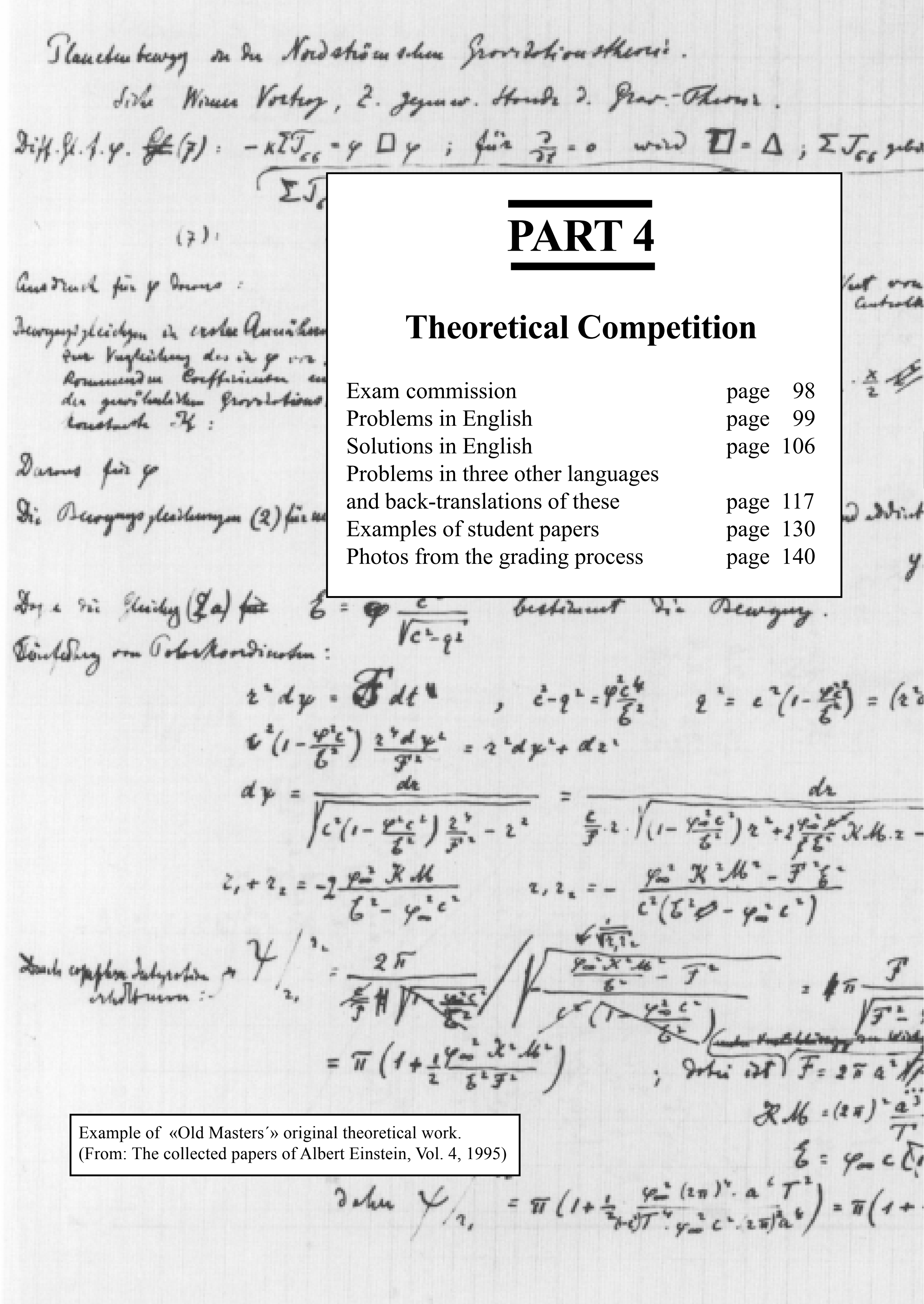
The last preparations

The problem for the experimental competition was discussed by the leaders and the organizers the evening before the exam. At this meeting the equipment was demonstrated for the first time (picture).

Photo: Børge Holme

After the meeting had agreed on the final text (in English), the problems had to be translated into the remaining 36 languages. One PC was available for each nation for the translation process (see picture below). The last nation finished their translation at about 4:30 a.m. in the morning, and the competition started at 0830. Busy time for the organizers! Examples of the different translations are given on the following pages.

Photo: Børge Holme



PART 4

Theoretical Competition

Example of «Old Masters´» original theoretical work.
(From: The collected papers of Albert Einstein, Vol. 4, 1995)

Per Chr. Hemmer
Chief examiner

Commission for the Theoretical Competiton:

Per Chr. Hemmer
Alex Hansen
Eivind Hiis Hauge
Kjell Mork
Kåre Olaussen

Norwegian University of Science and Technology, Trondheim

&

Torgeir Engeland
Yuri Galperin
Anne Holt
Asbjørn Kildal
Leif Veseth

University of Oslo

27th INTERNATIONAL PHYSICS OLYMPIAD
OSLO, NORWAY

THEORETICAL COMPETITION
JULY 2 1996

Time available: 5 hours

READ THIS FIRST :

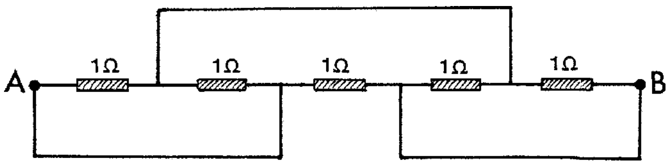
1. Use only the pen provided
2. Use only the marked side of the paper
3. Each problem should be answered on separate sheets
4. In your answers please use primarily equations and numbers, and as little *text* as possible
5. Write at the top of *every* sheet in your report:
   - Your candidate number (IPhO identification number)
   - The problem number and section identification, e.g. 2/a
   - Number each sheet consecutively
6. Write on the front page the total number of sheets in your report

This set of problems consists of 7 pages.

PROBLEM 1

(The five parts of this problem are unrelated)

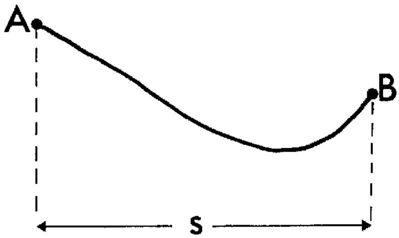
a) Five 1Ω resistances are connected as shown in the figure. The resistance in the conducting wires (fully drawn lines) is negligible.



100 Determine the resulting resistance R between A and B. *(1 point)*

---

b)



A skier starts from rest at point A and slides down the hill, without turning or braking. The friction coefficient is μ. When he stops at point B, his horizontal displacement is s. What is the height difference h between points A and B? (The velocity of the skier is small so that the additional pressure on the snow due to the curvature can be neglected. Neglect also the friction of air and the dependence of μ on the velocity of the skier.) *(1.5 points)*

---

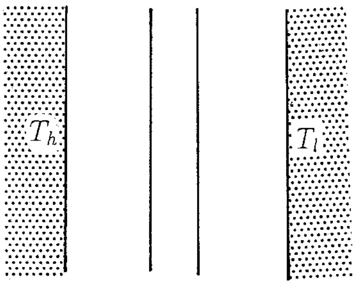
c) A thermally insulated piece of metal is heated under atmospheric pressure by an electric current so that it receives electric energy at a constant power P. This leads to an increase of the absolute temperature T of the metal with time t as follows:

$$T(t) = T_0\left[1 + a(t - t_0)\right]^{1/4}.$$

Here a, t_0 and T_0 are constants. Determine the heat capacity $C_p(T)$ of the metal (temperature dependent in the temperature range of the experiment). *(2 points)*

d) A black plane surface at a constant high temperature T_h is parallel to another black plane surface at a constant lower temperature T_l. Between the plates is vacuum.

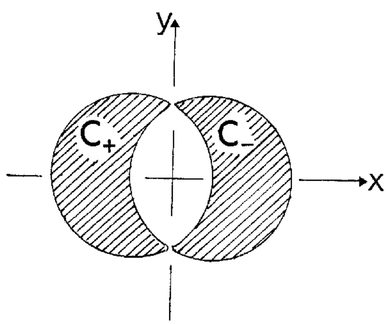
In order to reduce the heat flow due to radiation, a heat shield consisting of two thin black plates, thermally isolated from each other, is placed between the warm and the cold surfaces and parallel to these. After some time stationary conditions are obtained.



By what factor ξ is the stationary heat flow reduced due to the presence of the heat shield? Neglect end effects due to the finite size of the surfaces. *(1.5 points)*

---

e) Two straight and very long nonmagnetic conductors C_+ and C_-, insulated from each other, carry a current I in the positive and the negative z direction, respectively. The cross sections of the conductors (hatched in the figure) are limited by circles of diameter D in the x-y plane, with a distance $D/2$ between the centres. Thereby the resulting cross sections each have an area $(\frac{1}{12}\pi + \frac{1}{8}\sqrt{3})D^2$. The current in each conductor is uniformly distributed over the cross section.

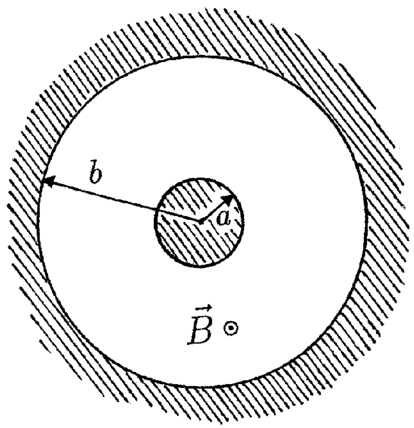


Determine the magnetic field $B(x,y)$ in the space between the conductors. *(4 points)*

PROBLEM 2

The space between a pair of coaxial cylindrical conductors is evacuated. The radius of the inner cylinder is a, and the inner radius of the outer cylinder is b, as shown in the figure below. The outer cylinder, called the anode, may be given a positive potential V relative to the inner cylinder. A static homogeneous magnetic field $\vec{B}$ parallel to the cylinder axis, directed out of the plane of the figure, is also present. Induced charges in the conductors are neglected.

We study the dynamics of electrons with rest mass m and charge $-e$. The electrons are released at the surface of the inner cylinder.



a) First the potential V is turned on, but $\vec{B} = 0$. An electron is set free with negligible velocity at the surface of the inner cylinder. Determine its speed v when it hits the anode. Give the answer both when a non-relativistic treatment is sufficient, and when it is not. *(1 point)*

For the remaining parts of this problem a non-relativistic treatment suffices.

b) Now $V = 0$, but the homogeneous magnetic field $\vec{B}$ is present. An electron starts out with an initial velocity $\vec{v}_0$ in the radial direction. For magnetic fields larger than a critical value B_c, the electron will not reach the anode. Make a sketch of the trajectory of the electron when B is slightly more than B_c. Determine B_c. *(2 points)*

From now on *both* the potential V *and* the homogeneous magnetic field $\vec{B}$ are present.

c) The magnetic field will give the electron a non-zero angular momentum L with respect to the cylinder axis. Write down an equation for the rate of change *dL/dt* of the angular momentum. Show that this equation implies that

$$L - keBr^2$$

is constant during the motion, where k is a definite pure number. Here r is the distance from the cylinder axis. Determine the value of k. *(3 points)*

d) Consider an electron, released from the inner cylinder with negligible velocity, that does not reach the anode, but has a maximal distance from the cylinder axis equal to r_m. Determine the speed v at the point where the radial distance is maximal, in terms of r_m. *(1 point)*

e) We are interested in using the magnetic field to regulate the electron current to the anode. For B larger than a critical magnetic field B_c, an electron, released with negligible velocity, will not reach the anode. Determine B_c.
(1 point)

f) If the electrons are set free by heating the inner cylinder an electron will in general have an initial nonzero velocity at the surface of the inner cylinder. The component of the initial velocity parallel to $\vec{B}$ is v_B, the components orthogonal to $\vec{B}$ are v_r (in the radial direction) and v_φ (in the azimuthal direction, i.e. orthogonal to the radial direction).

Determine for this situation the critical magnetic field B_c for reaching the anode. *(2 points)*

PROBLEM 3

In this problem we consider some gross features of the magnitude of mid-ocean tides on earth. We simplify the problem by making the following assumptions:

(i) The earth and the moon are considered to be an isolated system,
(ii) the distance between the moon and the earth is assumed to be constant,
(iii) the earth is assumed to be completely covered by an ocean,
(iv) the dynamic effects of the rotation of the earth around its axis are neglected, and
(v) the gravitational attraction of the earth can be determined as if all mass were concentrated at the centre of the earth.

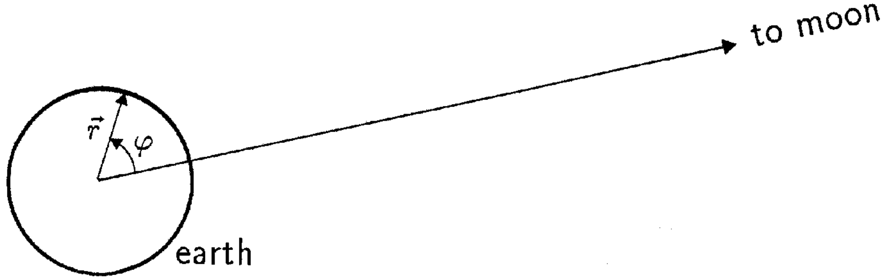
The following data are given:

Mass of the earth: $M = 5.98 \cdot 10^{24}$ kg
Mass of the moon: $M_m = 7.3 \cdot 10^{22}$ kg
Radius of the earth: $R = 6.37 \cdot 10^{6}$ m
Distance between centre of the earth and centre of the moon:
$L = 3.84 \cdot 10^{8}$ m
The gravitational constant: $G = 6.67 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2}.

a) The moon and the earth rotate with angular velocity ω about their common centre of mass, C. How far is C from the centre of the earth? (Denote this distance by l.)

Determine the numerical value of ω. *(2 points)*

We now use a frame of reference that is co-rotating with the moon and the center of the earth around C. In this frame of reference the shape of the liquid surface of the earth is static.



In the plane P through C and orthogonal to the axis of rotation the position of a point mass on the liquid surface of the earth can be described by polar coordinates r, φ as shown in the figure. Here r is the distance from the centre of the earth.

We will study the shape

$$r(\varphi) = R + h(\varphi)$$

of the liquid surface of the earth in the plane P.

b) Consider a mass point (mass m) on the liquid surface of the earth (in the plane P). In our frame of reference it is acted upon by a centrifugal force and by gravitational forces from the moon and the earth. Write down an expression for the potential energy corresponding to these three forces. 105

Note: Any force $F(r)$, radially directed with respect to some origin, is the negative derivative of a spherically symmetric potential energy $V(r)$: $F(r) = -V'(r)$. *(3 points)*

c) Find, in terms of the given quantities M, M_m, etc, the approximate form $h(\varphi)$ of the tidal bulge. What is the difference <u>in meters</u> between high tide and low tide in this model?

You may use the approximate expression

$$\frac{1}{\sqrt{1 + a^2 - 2a\cos\theta}} \approx 1 + a\cos\theta + \tfrac{1}{2}a^2(3\cos^2\theta - 1),$$

valid for a much less than unity.

In this analysis make simplifying approximations whenever they are reasonable. *(5 points)*

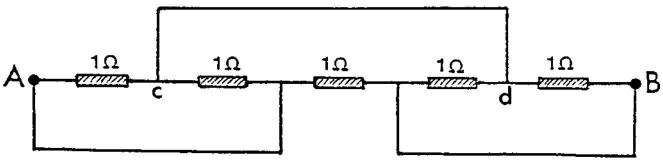
OSLO 1996
27th INTERNATIONAL
PHYSICS OLYMPIAD

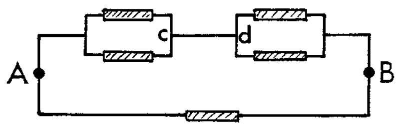
27th INTERNATIONAL PHYSICS OLYMPIAD
OSLO, NORWAY

THEORETICAL COMPETITION
JULY 2 1996

Solution Problem 1

a) The system of resistances can be redrawn as shown in the figure:





The equivalent drawing of the circuit shows that the resistance between point c and point A is 0.5Ω, and the same between point d and point B. The resistance between points A and B thus consists of two connections in parallel: the direct 1Ω connection and a connection consisting of two 0.5Ω resistances in series, in other words two parallel 1Ω connections. This yields

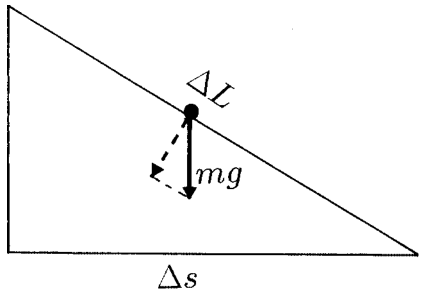
$$R = \underline{\underline{0.5\ \Omega}}\ .$$

b) For a sufficiently short horizontal displacement Δs the path can be considered straight. If the corresponding length of the path element is ΔL, the friction force is given by

$$\mu\, mg\frac{\Delta s}{\Delta L}$$

and the work done by the friction force equals force times displacement:

$$\mu\ mg\frac{\Delta s}{\Delta L}\cdot \Delta L = \mu\ mg\Delta s.$$



Adding up, we find that along the whole path the total work done by friction forces i $\mu\ \ mg\, s$. By energy conservation this must equal the decrease $mg\,h$ in potential energy of the skier. Hence

$$h = \underline{\underline{\mu s}}.$$

---

c) Let the temperature increase in a small time interval dt be dT. During this time interval the metal receives an energy $P\,dt$.

The heat capacity is the ratio between the energy supplied and the temperature increase:

$$C_p = \frac{Pdt}{dT} = \frac{P}{dT/dt}.$$

The experimental results correspond to

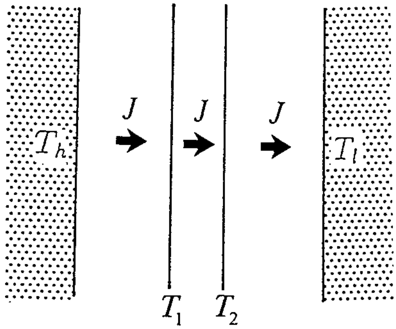
$$\frac{dT}{dt} = \frac{T_0}{4}a[1 + a(t - t_0)]^{-3/4} = T_0\frac{a}{4}\left(\frac{T_0}{T}\right)^3.$$

Hence

$$C_p = \frac{P}{dT/dt} = \underline{\frac{4P}{aT_0^{\,4}}T^3}.$$

(*Comment*: At low, but not extremely low, temperatures heat capacities of metals follow such a T^3 law.)

d)



Under stationary conditions the net heat flow is the same everywhere:

$$J = \sigma(T_h^4 - T_1^4)$$
$$J = \sigma(T_1^4 - T_2^4)$$
$$J = \sigma(T_2^4 - T_l^4)$$

Adding these three equations we get

$$3J = \sigma(T_h^4 - T_l^4) = J_0,$$

where J_0 is the heat flow in the absence of the heat shield. Thus $\xi = J/J_0$ takes the value

$$\xi = \underline{\underline{1/3}}.$$

---

e) The magnetic field can be determined as the superposition of the fields of two *cylindrical* conductors, since the effects of the currents in the area of intersection cancel. Each of the cylindrical conductors must carry a larger current I', determined so that the fraction I of it is carried by the actual cross section (the moon-shaped area). The ratio between the currents I and I' equals the ratio between the cross section areas:

$$\frac{I}{I'} = \frac{(\frac{\pi}{12} + \frac{\sqrt{3}}{8})D^2}{\frac{\pi}{4}D^2} = \frac{2\pi + 3\sqrt{3}}{6\pi}.$$

Inside one cylindrical conductor carrying a current I' Ampère's law yields at a distance r from the axis an azimuthal field

$$B_\phi = \frac{\mu_0}{2\pi r}\frac{I'\pi r^2}{\frac{\pi}{4}D^2} = \frac{2\mu_0 I' r}{\pi D^2}.$$

The cartesian components of this are

$$B_x = -B_\phi \frac{y}{r} = -\frac{2\mu_0 I' y}{\pi D^2}; \qquad B_y = B_\phi \frac{x}{r} = \frac{2\mu_0 I' x}{\pi D^2}.$$

For the superposed fields, the currents are $\pm I'$ and the corresponding cylinder axes are located at $x = \mp D/4$.

The two x-components add up to zero, while the y-components yield

$$B_y = \frac{2\mu_0}{\pi D^2}[I'(x + D/4) - I'(x - D/4)] = \frac{\mu_0 I'}{\pi D} = \underline{\underline{\frac{6\mu_0 I}{(2\pi + 3\sqrt{3})D}}},$$

i.e., a *constant* field. The direction is along the positive *y*-axis.

Solution Problem 2

a) The potential energy gain eV is converted into kinetic energy. Thus

$$\tfrac{1}{2} m v^2 = eV \qquad \text{(non-relativistically)}$$

$$\frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = eV \qquad \text{(relativistically).}$$

Hence

$$v = \begin{cases} \sqrt{2eV/m} & \text{(non - relativistically)} \\ c\sqrt{1 - \left(\dfrac{mc^2}{mc^2 + eV}\right)^2} & \text{(relativistically).} \end{cases} \qquad (1)$$

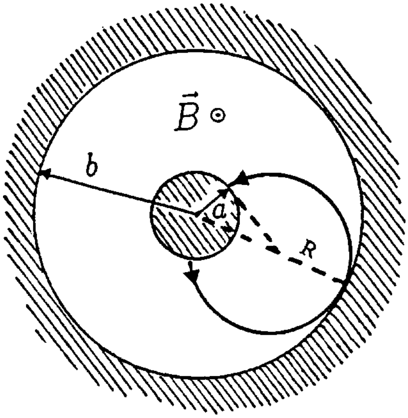
b) When $V = 0$ the electron moves in a homogeneous static magnetic field. The magnetic Lorentz force acts orthogonal to the velocity and the electron will move in a circle. The initial velocity is tangential to the circle.

The radius R of the orbit (the "cyclotron radius") is determined by equating the centripetal force and the Lorentz force:

$$eBv_0 = \frac{mv_0^2}{R},$$

i.e.

$$B = \frac{mv_0}{eR}. \qquad (2)$$



From the figure we see that in the critical case the radius R of the circle satisfies

$$\sqrt{a^2 + R^2} = b - R$$

By squaring we obtain

$$a^2 + R^2 = b^2 - 2bR + R^2,$$

i.e.

$$R = (b^2 - a^2)/2b.$$

Insertion of this value for the radius into the expression (2) gives the critical field

$$B_c = \frac{mv_0}{eR} = \underline{\underline{\frac{2bmv_0}{(b^2 - a^2)e}}}.$$

c) The change in angular momentum with time is produced by a torque. Here the azimuthal component F_ϕ of the Lorentz force $\vec{F} = (-e)\vec{B}\times\vec{v}$ provides a torque $F_\phi r$. It is only the radial component $v_r = dr/dt$ of the velocity that provides an azimuthal Lorentz force. Hence

$$\underline{\underline{\frac{dL}{dt} = eBr\frac{dr}{dt}}},$$

which can be rewritten as

$$\frac{d}{dt}(L - \frac{eBr^2}{2}) = 0.$$

Hence

$$C = \underline{\underline{L - \tfrac{1}{2} eBr^2}} \qquad (3)$$

is constant during the motion. The dimensionless number k in the problem text is thus $k = \underline{1/2}$.

d) We evaluate the constant C, equation (3), at the surface of the inner cylinder and at the maximal distance r_{m}:

$$0 - \tfrac{1}{2} eBa^2 = mvr_m - \tfrac{1}{2} eBr_m^2$$

which gives

$$\underline{v = \frac{eB(r_m^2 - a^2)}{2mr_m}}. \qquad (4)$$

Alternative solution: One may first determine the electric potential $V(r)$ as function of the radial distance. In cylindrical geometry the field falls off inversely proportional to r, which requires a logarithmic potential, $V(s) = c_1 \ln r + c_2$. When the two constants are determined to yield $V(a) = 0$ and $V(b) = V$ we have

$$V(r) = V \frac{\ln(r/a)}{\ln(b/a)}.$$

The gain in potential energy, $sV(r_m)$, is converted into kinetic energy:

$$\frac{1}{2} mv^2 = eV \frac{\ln(r_m/a)}{\ln(b/a)}.$$

Thus

$$v = \sqrt{\frac{2eV}{m} \frac{\ln(r_m/a)}{\ln(b/a)}}. \qquad (5)$$

(4) and (5) seem to be different answers. This is only apparent since r_m is not an independent parameter, but determined by B and V so that the two answers are identical.

e) For the critical magnetic field the maximal distance r_m equals b, the radius of the outer cylinder, and the speed at the turning point is then

$$v = \frac{eB(b^2 - a^2)}{2mb}.$$

Since the Lorentz force does no work, the corresponding kinetic energy $\frac{1}{2}mv^2$ equals eV (question a):

$$v = \sqrt{2eV/m}$$

The last two equations are consistent when

$$\frac{eB(b^2 - a^2)}{2mb} = \sqrt{2eV/m}.$$

The critical magnetic field for current cut-off is therefore

$$\underline{\underline{B_c = \frac{2b}{b^2 - a^2}\sqrt{\frac{2mV}{e}}}}.$$

f) The Lorentz force has no component parallel to the magnetic field, and consequently the velocity component v_B is constant under the motion. The corresponding displacement parallel to the cylinder axis has no relevance for the question of reaching the anode.

Let v denote the final azimuthal speed of an electron that barely reaches the anode. Conservation of energy implies that

$$\tfrac{1}{2}m(v_B^2 + v_\phi^2 + v_r^2) + eV = \tfrac{1}{2}m(v_B^2 + v^2),$$

giving

$$v = \sqrt{v_r^2 + v_\phi^2 + 2eV/m}. \tag{6}$$

Evaluating the constant C in (3) at both cylinder surfaces for the critical situation we have

$$mv_\phi a - \tfrac{1}{2}eB_c a^2 = mvb - \tfrac{1}{2}eB_c b^2.$$

Insertion of the value (6) for the velocity v yields the critical field

$$B_c = \frac{2m(vb - v_\phi a)}{e(b^2 - a^2)} = \frac{2mb}{e(b^2 - a^2)}\left[\sqrt{v_r^2 + v_\phi^2 + 2eV/m} - v_\phi a/b\right].$$

Solution Problem 3

a) With the centre of the earth as origin, let the centre of mass C be located at $\vec{l}$. The distance l is determined by

$$M\,l = M_m\,(L - l),$$

which gives

$$l = \frac{M_m}{M + M_m} L = \underline{4.63 \cdot 10^6\ \text{m},} \tag{1}$$

less than R, and thus inside the earth.

The centrifugal force must balance the gravitational attraction between the moon and the earth:

$$M\omega^2 l = G\frac{MM_m}{L^2},$$

which gives

$$\omega = \sqrt{\frac{GM_m}{L^2 l}} = \underline{\underline{\sqrt{\frac{G(M + M_m)}{L^3}}}} = \underline{\underline{2.67 \cdot 10^{-6}\,\text{s}^{-1}}}. \tag{2}$$

(This corresponds to a period $2\pi/\omega = 27.2$ days.) We have used (1) to eliminate l.

b) The potential energy of the mass point m consists of three contributions:

(1) Potential energy because of rotation (in the rotating frame of reference, see the problem text),

$$-\frac{1}{2} m\omega^2 r_1^2,$$

where $\vec{r}_1$ is the distance from C. This corresponds to the centrifugal force $m\omega^2 r_1$, directed outwards from C.

(2) Gravitational attraction to the earth,

$$-G\frac{mM}{r}.$$

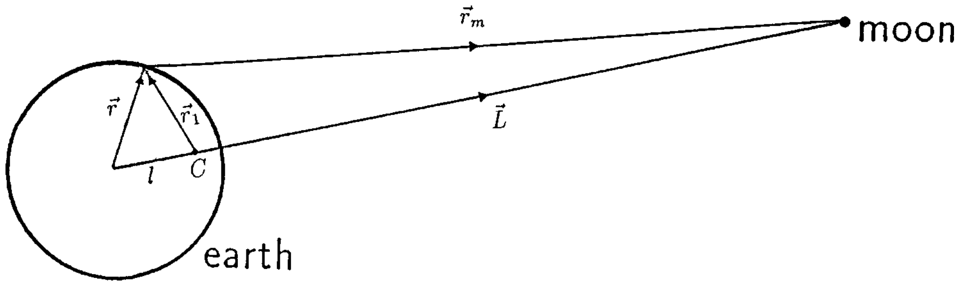
(3) Gravitational attraction to the moon,

$$-G\frac{mM_m}{|\vec{r}_m|},$$

where $\vec{r}_m$ is the distance from the moon.

Describing the position of m by polar coordinates $r;\ \phi$ in the plane orthogonal to the axis of rotation (see figure), we have

$$\vec{r}_1^{\,2} = (\vec{r}-\vec{l}\,)^2 = r^2 - 2rl\cos\phi + l^2.$$



Adding the three potential energy contributions, we obtain

$$V(\vec{r}) = -\frac{1}{2}m\omega^2(r^2 - 2rl\cos\phi + l^2) - G\frac{mM}{r} - G\frac{mM_m}{|\vec{r}_m|}. \quad (3)$$

Here l is given by (1) and

$$|\vec{r}_m| = \sqrt{(\vec{L}-\vec{r})^2} = \sqrt{L^2 - 2\vec{L}\vec{r} + r^2} = L\sqrt{1+(r/L)^2 - 2(r/L)\cos\phi}.$$

c) Since the ratio $r/L = a$ is very small, we may use the expansion

$$\frac{1}{\sqrt{1+a^2-2a\cos\phi}} = 1 + a\cos\phi + a^2\,\tfrac{1}{2}(3\cos^2\phi - 1).$$

Insertion into the expression (3) for the potential energy gives

$$V(r,\phi)/m = -\tfrac{1}{2}\omega^2 r^2 - \frac{GM}{r} - \frac{GM_m r^2}{2L^3}(3\cos^2\phi - 1), \quad (4)$$

apart from a constant. We have used that

$$m\omega^2 rl\cos\phi - GmM_m\frac{r}{L^2}\cos\phi = 0,$$

when the value of ω_2, equation (2), is inserted.

The form of the liquid surface is such that a mass point has the same energy V *everywhere on the surface*. (This is equivalent to requiring no net force tangential to the surface.) Putting

$$r = R + h,$$

where the tide h is much smaller than R, we have approximately

$$\frac{1}{r} = \frac{1}{R+h} = \frac{1}{R} \cdot \frac{1}{1+(h/R)} \cong \frac{1}{R}(1-\frac{h}{R}) = \frac{1}{R} - \frac{h}{R^2},$$

as well as

$$r^2 = R^2 + 2Rh + h^2 \cong R^2 + 2Rh.$$

Inserting this, and the value (2) of ω into (4), we have

$$V(r,\phi)/m = -\frac{G(M+M_m)R}{L^3}h + \frac{GM}{R^2}h - \frac{GM_m r^2}{2L^3}(3\cos^2\phi - 1), \qquad (5)$$

again apart from a constant.

The magnitude of the first term on the right-hand side of (5) is a factor

$$\frac{(M+M_m)}{M}\left(\frac{R}{L}\right)^3 \cong 10^{-5}$$

smaller than the second term, thus negligible. If the remaining two terms in equation (5) compensate each other, i.e.,

$$h = \frac{M_m r^2 R^2}{2ML^3}(3\cos^2\phi - 1),$$

then the mass point m has the same energy everywhere on the surface. Here r^2 can safely be approximated by R^2, giving the tidal bulge

$$\underline{\underline{h = \frac{M_m R^4}{2ML^3}(3\cos^2\phi - 1).}}$$

The largest value $h_{\max} = M_m R^4/ML^3$ occurs for $\phi = 0$ or π, in the direction of the moon or in the opposite direction, while the smallest value

$$h_{\min} = -M_m R^4 / 2ML^3$$

corresponds to $\phi = \pi/2$ or $3\pi/2$.

The difference between high tide and low tide is therefore

$$h_{\max} - h_{\min} = \frac{3M_m R^4}{2ML^3} = \underline{\underline{0.54\text{m.}}}$$

(The values for high and low tide are determined up to an additive constant, but the difference is of course independent of this.)

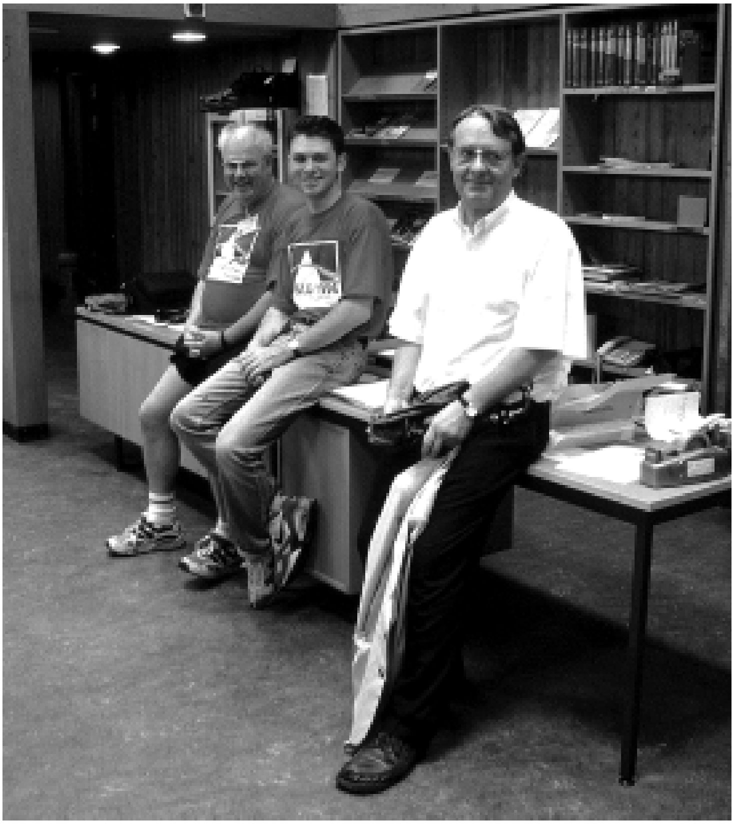


Photo: Arnt Inge Vistnes

Here we see the Exam Officer, Michael Peachey (in the middle), with his helper Rod Jory (at the left), both from Australia, as well as the Chief examiner, Per Chr. Hemmer. The picture was taken in a silent moment during the theory examination. Michael and Rod had a lot *of experience from the 1995 IPhO in Canberra, so their help was very effective and highly appreciated!*