# Theoretical Competition

Monday, July 2nd, 2001

Please read this first:

1. The time available is 5 hours for the theoretical competition.
2. Use only the pen provided.
3. Use only the front side of the paper.
4. Begin each part of the problem on a separate sheet.
5. For each question, in addition to the **blank sheets** where you may write, there is an **answer form** where you *must* summarize the results you have obtained. Numerical results should be written with as many digits as are appropriate to the given data.
6. Write on the blank sheets of paper whatever you consider is required for the solution of the question. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
7. Fill in the boxes at the top of each sheet of paper used by writing your **Country No** and **Country Code,** your student number (**Student No**), the number of the question **(Question No)**, the progressive number of each sheet **(Page No),** and the total number of blank sheets used for each question **(Total No of pages).** Write the question number and the section letter of the part you are answering at the top of each sheet. If you use some blank sheets of paper for notes that you do not wish to be marked, put a large X across the entire sheet and do not include it in your numbering.
8. At the end of the exam, arrange all sheets for each problem *in the following order;*
   - answer form
   - used sheets in order
   - the sheets you do not wish to be marked
   - unused sheets and the printed question
   Place the papers inside the envelope and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room.

Question 1

1a) KLYSTRON

Klystrons are devices used for amplifying very high-frequency signals. A klystron basically consists of two identical pairs of parallel plates (cavities) separated by a distance b, as shown in the figure.
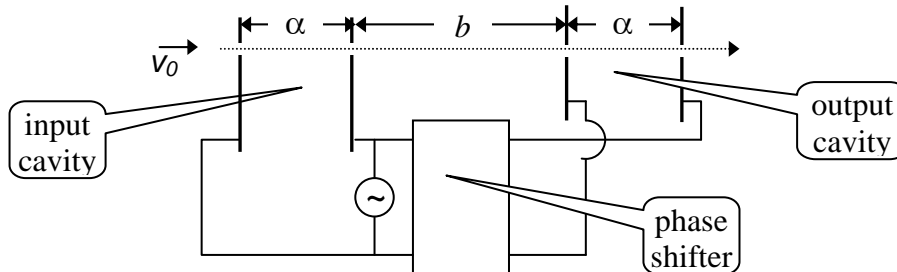
An electron beam with an initial speed v_0 traverses the entire system, passing through small holes in the plates. The high-frequency voltage to be amplified is applied to both pairs of plates with a certain phase difference (where period T corresponds to 2π phase) between them, producing horizontal, alternating electric fields in the cavities. The electrons entering the input cavity when the electric field is to the right are retarded and vice versa, so that the emerging electrons form bunches at a certain distance. If the output cavity is placed at the bunching point, the electric field in this cavity will absorb power from the beam provided that its phase is appropriately chosen. Let the voltage signal be a square wave with period $T=1.0 \times 10^{-9}$ s, changing between $V=\pm 0.5$ volts. The initial velocity of the electrons is $v_0=2.0 \times 10^6$ m/s and the charge to mass ratio is $e/m=1.76 \times 10^{11}$ C/kg. The distance α is so small that the transit time in the cavities can be neglected. Keeping 4 significant figures, calculate;

a) the distance b, where the electrons bunch. Copy your result onto the **answer form.** [1.5 pts]

b) the necessary phase difference to be provided by the phase shifter. Copy your result onto the **answer form.** [1.0 pts]

1b) INTERMOLECULAR DISTANCE

Let d_L and d_V represent the average distances between molecules of water in the liquid phase and in the vapor phase, respectively. Assume that both phases are at 100 °C and atmospheric pressure, and the vapor behaves like an ideal gas. Using the following data, calculate the ratio d_V /d_L and copy your result onto the **answer form.** [2.5 pts]

Density of water in liquid phase: $\rho_L=1.0 \times 10^3$ kg/m^3,

Molar mass of water: $M=1.8 \times 10^{-2}$ kg/mol
Atmospheric pressure: $P_a=1.0 \times 10^5$ N/m^2
Gas constant: $R=8.3$ J/mol. K
Avagadro's number: $N_A=6.0 \times 10^{23}$ /mol

1c) SIMPLE SAWTOOTH SIGNAL GENERATOR

A sawtooth voltage waveform V_0 can be obtained across the capacitor C in Fig. 1. R is a variable resistor, V_i is an ideal battery, and SG is a spark gap consisting of two electrodes with an adjustable distance between them. When the voltage across the electrodes exceeds the firing voltage V_f, the air between the electrodes breaks down, hence the gap becomes a short circuit and remains so until the voltage across the gap becomes very small.
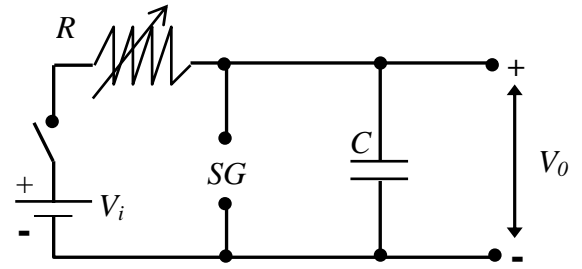
Figure 1

a) Draw the voltage waveform V_0 versus time t, after the switch is closed. *[0.5 pts]*

b) What condition must be satisfied in order to have an almost linearly varying sawtooth voltage waveform V_0? Copy your result onto the **answer form**. *[0.2 pts]*

c) Provided that this condition is satisfied, derive a simplified expression for the period T of the waveform. Copy your result onto the **answer form.** *[0.4 pts]*

d) What should you vary(R and/or SG) to change the period only? Copy your result onto the **answer form.** *[0.2 pts]*

e) What should you vary (R and/or SG) to change the amplitude only? Copy your result onto the **answer form.** *[0.2 pts]*

f) You are given an additional, adjustable DC voltage supply. Design and draw a new circuit indicating the terminals where you would obtain the voltage waveform V_0' described in Fig. 2. *[1.0 pts]*
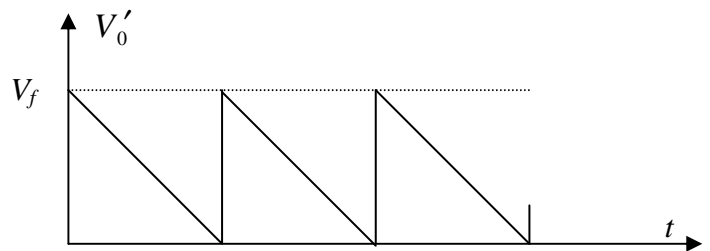
Figure 2

1d) ATOMIC BEAM

An atomic beam is prepared by heating a collection of atoms to a temperature T and allowing them to emerge horizontally through a small hole (of atomic dimensions) of diameter D in one side of the oven. Estimate the diameter of the beam after it has traveled a horizontal length L along its path. The mass of an atom is M. Copy your result onto the **answer form.** *[2.5 pts]*

Diameter $= D$

L

Oven at temperature T

Atoms of mass M
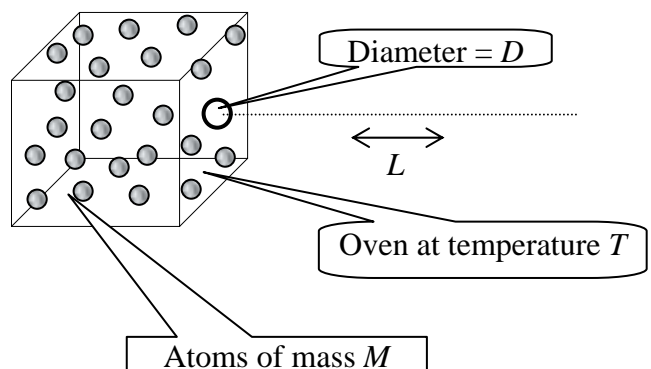
Question 2

BINARY STAR SYSTEM

a) It is well known that most stars form binary systems. One type of binary system consists of an ordinary star with mass m_0 and radius R, and a more massive, compact neutron star with mass M, rotating around each other. In all the following ignore the motion of the earth. Observations of such a binary system reveal the following information:

- The maximum angular displacement of the ordinary star is $\Delta\theta$, whereas that of the neutron star is $\Delta\phi$ (see Fig. 1).
- The time it takes for these maximum displacements is τ.
- The radiation characteristics of the ordinary star indicate that its surface temperature is T and the radiated energy incident on a unit area on earth's surface per unit time is P.
- The calcium line in this radiation differs from its normal wavelength λ_0 by an amount $\Delta\lambda$, due only to the gravitational field of the ordinary star. (For this calculation the photon can be considered to have an effective mass of $h/c\lambda$.)
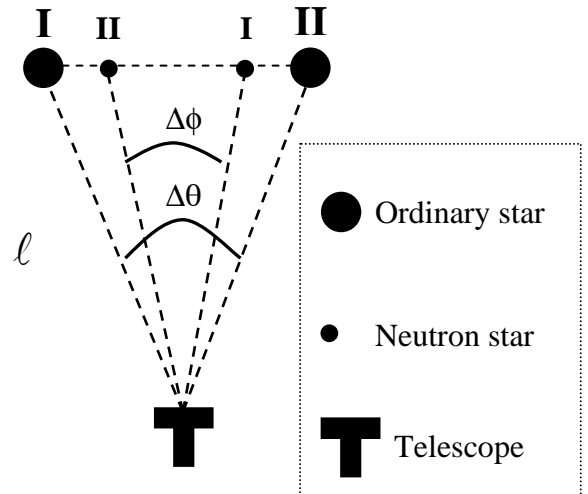
Fig. 1

Find an expression for the distance ℓ from earth to this system, only in terms of the observed quantities and universal constants. Copy your result onto the **answer form**. *[7 pts]*

b) Assume that $M \gg m_0$, so that the ordinary star is basically rotating around the neutron star in a circular orbit of radius r_0. Assume that the ordinary star starts emitting gas toward the neutron star with a speed v_0, relative to the ordinary star (see Fig. 2). Assuming that the neutron star is the dominant gravitational force in this problem and neglecting the orbital changes of the ordinary star find the distance of closest approach r_f shown in Fig. 2. Copy your result onto the **answer form**. *[3pts]*
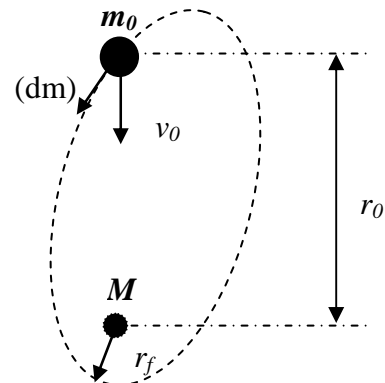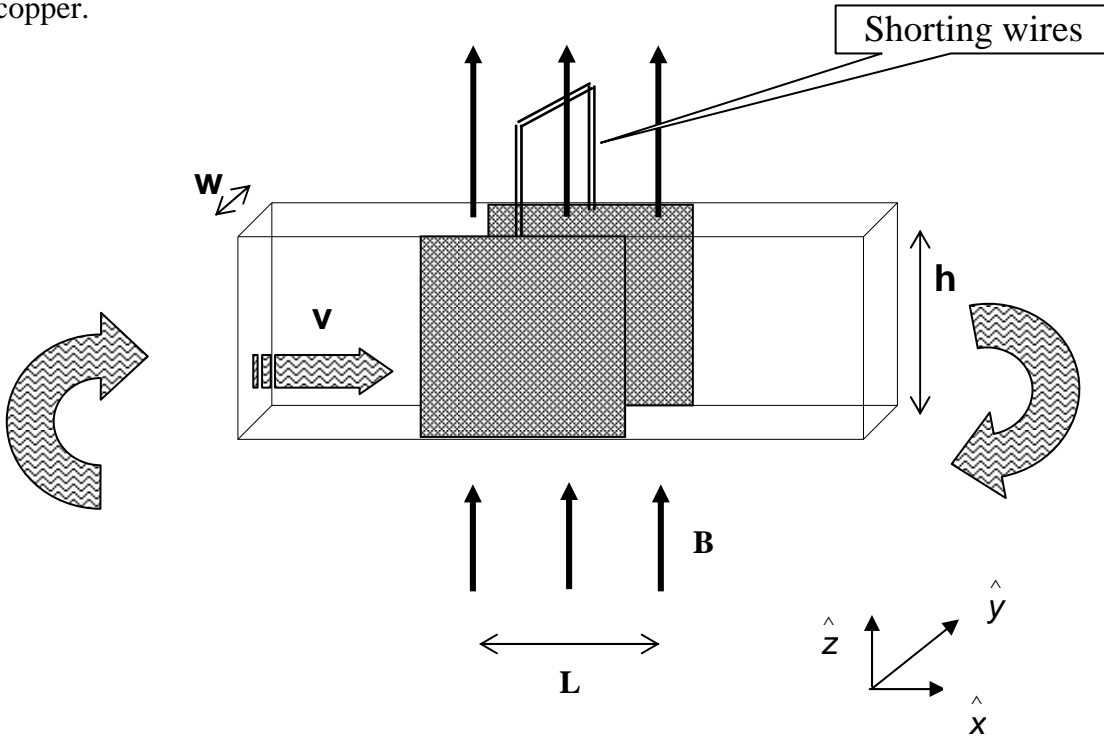
Fig. 2

Question 3

MAGNETOHYDRODYNAMIC (MHD) GENERATOR

A horizontal rectangular plastic pipe of width w and height h, which closes upon itself, is filled with mercury of resistivity ρ. An overpressure P is produced by a turbine which drives this fluid with a constant speed v_0. The two opposite vertical walls of a section of the pipe with length L are made of copper.

The motion of a real fluid is very complex. To simplify the situation we assume the following:
- Although the fluid is viscous, its speed is uniform over the entire cross section.
- The speed of the fluid is always proportional to the net external force acting upon it.
- The fluid is incompressible.

These walls are electrically shorted externally and a uniform, magnetic field **B** is applied vertically upward only in this section. The set up is illustrated in the figure above, with the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ to be used in the solution.

a) Find the force acting on the fluid due to the magnetic field (in terms of L, B, h, w, ρ and the new velocity v) [2.0 pts]
b) Derive an expression for the new speed v of the fluid (in terms of v_0, P, L, B and ρ) after the magnetic field is applied. [3.0 pts]
c) Derive an expression for the additional power that must be supplied by the turbine to increase the speed to its original value v_0. Copy your result onto the **answer form.** [2.0 pts]
d) Now the magnetic field is turned off and mercury is replaced by water flowing with speed v_0. An electromagnetic wave with a single frequency is sent along the section with length L in the direction of the flow. The refractive index of water is n, and $v_0 \ll c$. Derive an expression for the contribution of the fluid's motion to the phase difference between the waves entering and leaving section L. Copy your result onto the **answer form.** [3.0 pts]

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ANSWER FORM

1A

a)

$b =$

b)

Phase difference=

1B

$\dfrac{d_V}{d_L} =$

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1C

b)

c)

$T=$

d)

e)

1D

New diameter of the beam =

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ANSWER FORM

2a)

$\ell =$

2b)

$r_f =$

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ANSWER FORM

3a)

3b)

v =

3c)

Power =

3d)

Phase difference =

Experimental Competition

Saturday, June 30th, 2001

Please read this first:

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2. Use only the pen provided.
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ROTATING LIQUID

This experiment consists of three basic parts:
1. investigation of the profile of the rotating liquid's surface and the determination of the acceleration due to gravity,
2. investigation of the rotating liquid as an optical system,
3. determination of the refractive index of the liquid.

When a cylindrical container filled with a liquid rotates about the vertical axis passing through its center with a uniform angular velocity ω, the liquid's surface becomes parabolic (see Figure 1). At equilibrium, the tangent to the surface at the point P(x, y) makes an angle θ with the horizontal such that

$$tan\,\theta = \frac{\omega^2 x}{g} \qquad for\ |x| \le R \tag{1}$$

where R is the radius of the container and g is the acceleration due to gravity.

It can further be shown that for $\omega<\omega_{max}$ (where ω_{max} is the angular speed at which the center of the rotating liquid touches the bottom of the container)

$$at\ x=x_0=\frac{R}{\sqrt{2}},\ y(x_0)=h_0 \tag{2}$$

that is; the height of the rotating liquid is the same as if it were not rotating.

The profile of the rotating liquid's surface is a parabola defined by the equation

$$y = y_0 + \frac{x^2}{4C} \tag{3}$$

where the vertex is at V($0, y_0$) and the focus is at F($0, y_0+C$). When optical rays parallel to the axis of symmetry (optical axis) reflect at the parabolic surface, they all focus at the point F (see Fig.1).

Apparatus

- A cylindrical rigid plastic cup containing liquid glycerin. Millimetric scales are attached to the bottom and the sidewall of this cup.
- A turntable driven by a small dc electric motor powered by a variable voltage supply, which controls the angular velocity.
- A transparent horizontal screen on which you can put transparent or semi-transparent millimetric scales. The location of the screen can be adjusted along the vertical and horizontal directions.
- A laser pointer mounted on a stand. The position of the pointer can be adjusted. The head of the pointer can be changed.
- Additional head for the laser pointer.
- A ruler.
- A highlighter pen.
- A stopwatch. Push the left button to reset, the middle button to select the mode, and the right button to start and stop the timing.
- Transmission gratings with 500 or 1000 lines/mm.
- Bubble level.
- Glasses.

IMPORTANT NOTES

- ***DO NOT LOOK DIRECTLY INTO THE LASER BEAM. BE AWARE THAT LASER LIGHT CAN ALSO BE DANGEROUS WHEN REFLECTED OFF A MIRROR-LIKE SURFACE. FOR YOUR OWN SAFETY USE THE GIVEN GLASSES.***

- *Throughout the whole experiment carefully handle the cup containing glycerin.*

- *The turntable has already been previously adjusted to be horizontal. Use bubble level only for horizontal alignment of the screen.*

- *Throughout the entire experiment you will observe several spots on the screen produced by the reflected and/or refracted beams at the various interfaces between the air, the liquid, the screen, and the cup. Be sure to make your measurements on the correct beam.*

- *In rotating the liquid change the speed of rotation gradually and wait for long enough times for the liquid to come into equilibrium before making any measurements.*

EXPERIMENT

PART 1: DETERMINATION of *g* USING a ROTATING LIQUID *[7.5 pts]*

- Derive Equation 1.
- Measure the height h_0 of the liquid in the container and the inner diameter $2R$ of the container.
- Insert the screen between the light source and the container. Measure the distance H between the screen and the turntable (see Figure 2).
- Align the laser pointer such that the beam points vertically downward and hits the surface of the liquid at a distance $x_0 = \dfrac{R}{\sqrt{2}}$ from the center of the container.
- Rotate the turntable slowly. Be sure that the center of the rotating liquid is not touching the bottom of the container.
- It is known that at $x_0 = \dfrac{R}{\sqrt{2}}$ the height of the liquid remains the same as the original height h_0, regardless of the angular speed ω. Using this fact and measurements of the angle θ of the surface at x_0 for various values of ω, perform an experiment to determine the gravitational acceleration g.
- Prepare tables of measured and calculated quantities for each ω.
- Produce the necessary graph to calculate g.
- Calculate the value of g and the experimental error in it
- Copy the values $2R, x_0, h_0, H$ and the experimental value of g and its error onto **the answer form.**

PART 2: OPTICAL SYSTEM

In this part of the experiment the rotating liquid will be treated as an image forming optical system. Since the curvature of the surface varies with the angular speed of rotation, the focal distance of this optical system depends on ω.

2a) Investigation of the focal distance *[5.5 pts]*

- Align the laser pointer such that the laser beam is directed vertically downward at the center of the container. Mark the point *P* where the beam strikes the screen. Thus the line joining this point to the center of the cup is the optical axis of this system (see Figure 2).
- Since the surface of the liquid behaves like a parabolic mirror, any incident beam parallel to the optical axis will pass through the focal point F on the optical axis after reflection.
- Adjust the speed of rotation to locate the focal point on the screen. Measure the angular speed of rotation ω and the distance *H* between the screen and the turntable.
- Repeat the above steps for different *H* values.
- Copy the measured values of **2R** and h_0 and the value of ω at each *H* onto the **answer form.**
- With the help of an appropriate graph of your data, find the relationship between the focal length and the angular speed. Copy your result onto the **answer form.**

2b) Analysis of the "image" (what you see on the screen) *[3.5 pts]*

In this part of the experiment the properties of the "image" produced by this optical system will be analyzed. To do so, follow the steps given below.

- Remove the head of the laser pointer by turning it counterclockwise.
- Mount the new head (provided in an envelope) by turning it clockwise. Now your laser produces a well defined shape rather than a narrow beam.
- Adjust the position of the laser pointer so that the beam strikes at about the center of the cup almost normally.
- Put a semitransparent sheet of paper on the horizontal screen, which is placed close to the cup, such that the laser beam does not pass through the paper, but the reflected beam does.
- Observe the size and the orientation of the "image" produced by the source beam and the beam reflected from the liquid when it is not rotating.
- Start the liquid rotating, and increase the speed of rotation gradually up to the maximum attainable speed while watching the screen. As ω increases you might observe different frequency ranges over which the properties of the "image" are drastically different. To describe these observations **complete the table** on the **answer form** by adding a row to this table for each such frequency range and fill it in by using the appropriate notations explained on that page.

PART 3: REFRACTIVE INDEX *[3.5 pts]*

In this part of the experiment the refractive index of the given liquid will be determined using a grating. When monochromatic light of wavelength λ is incident normally on a diffraction grating, the maxima of the diffraction pattern are observed at angles α_m given by the equation

$$m\lambda = d \sin\alpha_m \tag{4}$$

where, m is the order of diffraction and d is the distance between the rulings of the grating. In this part of the experiment a diffraction grating will be used to determine the wavelength of the laser light and the refractive index of the liquid (see Figure 3).

- Use the grating to determine the wavelength of the laser pointer. Copy your result onto the **answer form.**
- Immerse the grating perpendicularly into the liquid at the center of the cup.
- Align the laser beam such that it enters the liquid from the sidewall of the cup and strikes the grating normally.
- Observe the diffraction pattern produced on the millimetric scale attached to the cup on the opposite side. Make any necessary distance measurements.
- Calculate the refractive index n of the liquid by using your measurements. (Ignore the effect of the plastic cup on the path of the light.)
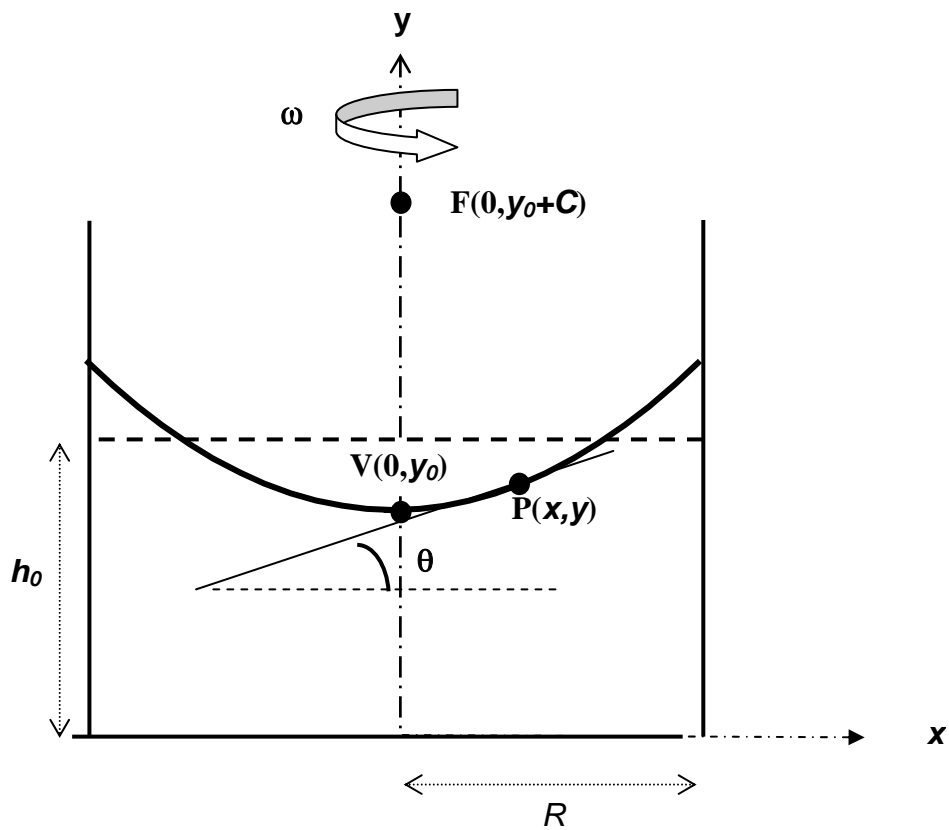- Copy the result of your experiment onto the **answer form.**

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Figure 1. Definitions of the bank angle θ at point P(x,y), the vertex V and the focus F for the parabolic surface produced by rotating the liquid, of initial height h_0 and radius R, at a constant angular speed ω around the y-axis.
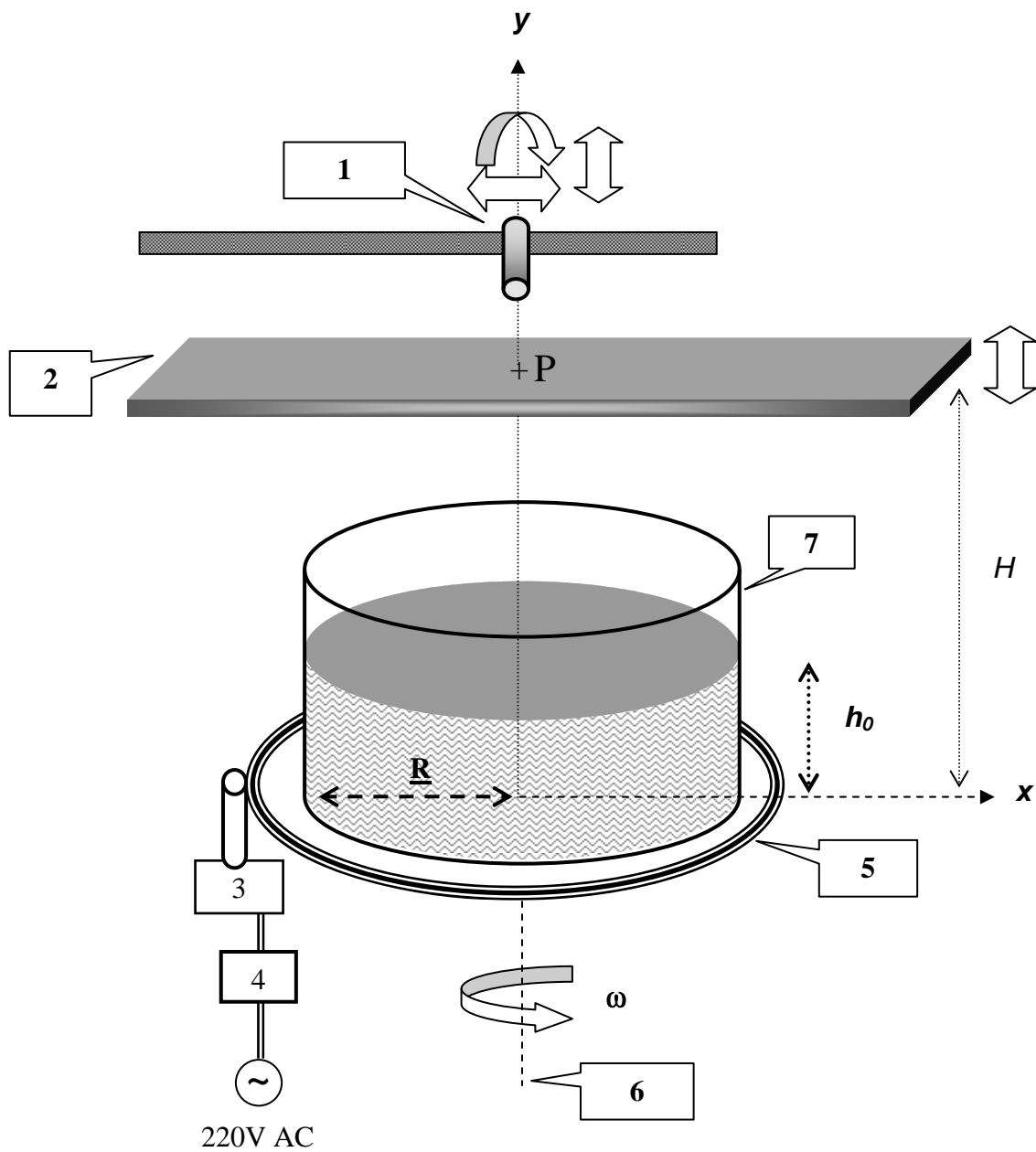
Figure 2 Experimental setup for parts 1 and 2.

1. Laser pointer on a stand, **2.** Transparent screen, **3.** Motor, **4**. Motor controller, **5**. Turntable, **6.** Axis of rotation, **7.** Cylindrical container.
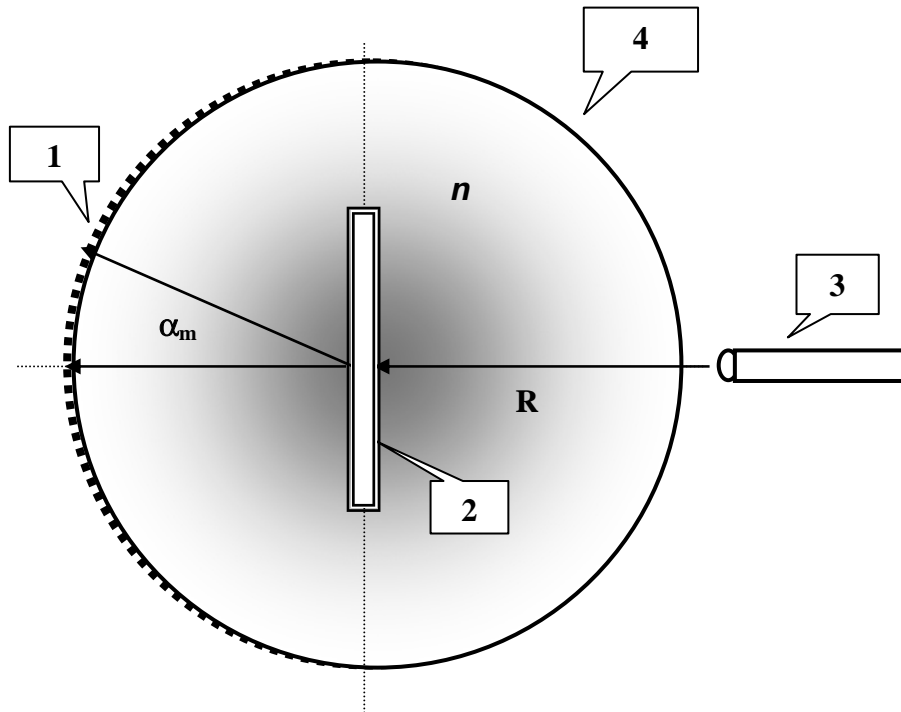
Figure 3 Top view of the grating in a liquid experiment.

1. Scaled sidewall, **2**. Grating on a holder, **3**. Laser pointer, **4**. Cylindrical container.

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ANSWER FORM

1) Determination of g using a rotating liquid

$2R$	x_0	h_0	H

Experimental value of g:

2a) Investigation of the focal distance

$2R$	h_0

H	ω

Relation between focal length and ω:

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2b) Analysis of the "image"

Use the appropriate notations explained below to describe what you see on the screen due to reflected beam

ω range: For the frequency ranges only approximate values are required.

Orientation (in comparison with the object beam as seen on the transparent screen):
Inverted   : **INV**
Erect   : **ER**

Variation of the size with increasing ω:
Increases   : **I**
Decreases   : **D**
No change : **NC**

For the frequency ranges you have found above:

Write "**R**" if the laser pointer is above the focal point.
Write "**V**" if the laser pointer is below the focal point.

ω Range	Orientation	Variation of the size	"image"
ω=0		■	

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3) Refractive index

Wavelength =

Experimental value for n =