37th International Physics Olympiad

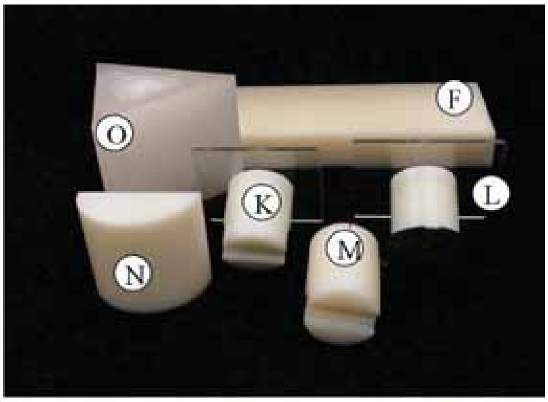
Singapore

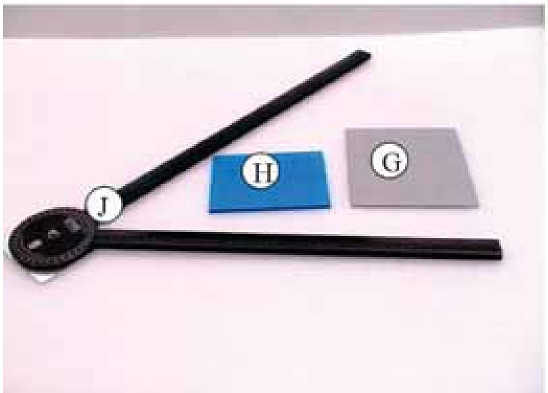
8 - 17 July 2006

Experimental Competition

Wed 12 July 2006

List of apparatus and materials





Label	Component	Quantity	Label	Component	Quantity
Ⓐ	Microwave transmitter	1	Ⓘ	Lattice structure in a black box	1
Ⓑ	Microwave receiver	1	Ⓙ	Goniometer	1
Ⓒ	Transmitter/receiver holder	2	Ⓚ	Prism holder	1
Ⓓ	Digital multimeter	1	Ⓛ	Rotating table	1
Ⓔ	DC power supply for transmitter	1	Ⓜ	Lens/reflector holder	1
Ⓕ	Slab as a "Thin film" sample	1	Ⓝ	Plano-cylindrical lens	1
Ⓖ	Reflector (silver metal sheet)	1	Ⓞ	Wax prism	2
Ⓗ	Beam splitter (blue Perspex)	1		Blu-Tack	1 pack
	Vernier caliper (provided separately)			30 cm ruler (provided separately)	

Caution:

- **The output power of the microwave transmitter is well within standard safety levels. Nevertheless, one should never look directly into the microwave horn at close range when the transmitter is on.**
- **Do not open the box containing the lattice Ⓘ.**
- **The wax prisms Ⓞ are fragile (used in Part 3).**

Note:

- ***It is important to note that the microwave receiver output (CURRENT) is proportional to the AMPLITUDE of the microwave.***
- ***Always use LO gain setting of the microwave receiver.***
- ***Do not change the range of the multimeter during the data collection.***
- ***Place the unused components away from the experiment to minimize interference.***
- ***Always use the component labels (Ⓐ, Ⓑ, Ⓒ,…) to indicate the components in all your drawings.***

The digital multimeter should be used with the two leads connected as shown in the diagram. You should use the “2m” current setting in this experiment.

Part 1: Michelson interferometer

1.1. Introduction

In a Michelson interferometer, a beam splitter sends an incoming electromagnetic (EM) wave along two separate paths, and then brings the constituent waves back together after reflection so that they superpose, forming an interference pattern. Figure 1.1 illustrates the setup for a Michelson interferometer. An incident wave travels from the transmitter to the receiver along two different paths. These two waves superpose and interfere at the receiver. The strength of signal at the receiver depends on the phase difference between the two waves, which can be varied by changing the optical path difference.

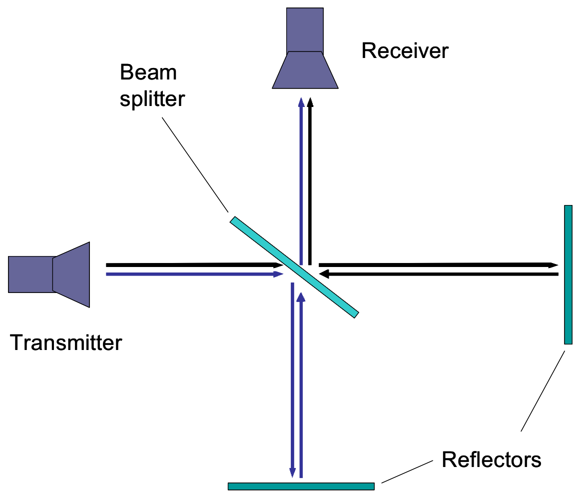


Figure 1.1: Schematic diagram of a Michelson interferometer.

1.2. List of components

1) Microwave transmitter Ⓐ with holder Ⓒ
2) Microwave receiver Ⓑ with holder Ⓒ
3) Goniometer Ⓙ
4) 2 reflectors: reflector Ⓖ with holder Ⓜ and thin film Ⓕ acting as a reflector.
5) Beam splitter Ⓗ with rotating table Ⓛ acting as a holder
6) Digital multimeter Ⓓ

1.3. Task: Determination of wavelength of the microwave [2 marks]

Using only the experimental components listed in Section 1.2, set up a Michelson interferometer experiment to determine the wavelength λ of the microwave in air. Record your data and determine λ in such a way that the uncertainty is ≤ 0.02 cm.

Note that the "thin film" is partially transmissive, so make sure you do not stand or move behind it as this might affect your results.

Part 2: "Thin film" interference

2.1. Introduction

A beam of EM wave incident on a dielectric thin film splits into two beams, as shown in Figure 2.1. Beam A is reflected from the top surface of the film whereas beam B is reflected from the bottom surface of the film. The superposition of beams A and B results in the so called thin film interference.

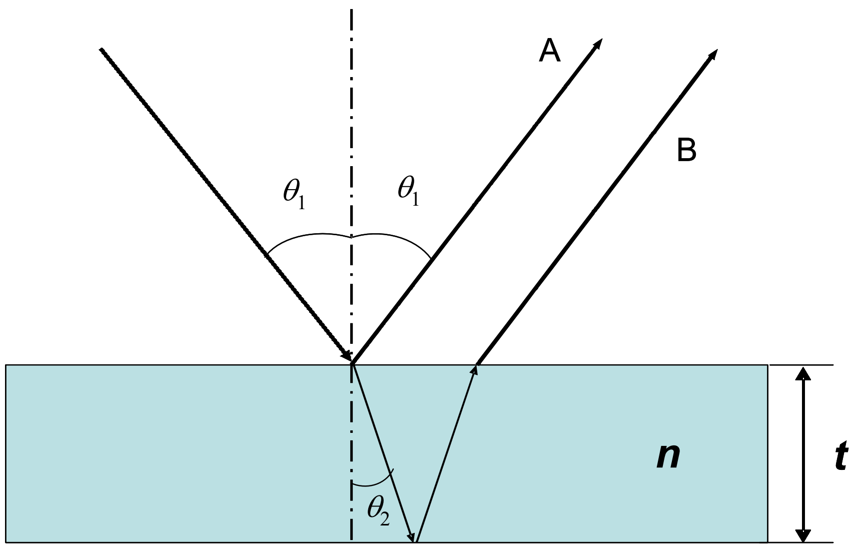


Figure 2.1: Schematic of thin film interference.

The difference in the optical path lengths of beam A and B leads to constructive or destructive interference. The resultant EM wave intensity I depends on the path difference of the two interfering beams which in turn depends on the angle of incidence, θ_1, of the incident beam, wavelength λ of the radiation, and the thickness t and refractive index n of the thin film. Thus, the refractive index n of the thin film can be determined from I-θ_1 plot, using values of t and λ.

2.2. List of components

1) Microwave transmitter Ⓐ with holder Ⓒ
2) Microwave receiver Ⓑ with holder Ⓒ
3) Plano-cylindrical lens Ⓝ with holder Ⓜ
4) Goniometer Ⓙ
5) Rotating table Ⓛ
6) Digital multimeter Ⓓ
7) Polymer slab acting as a "thin film" sample Ⓕ
8) Vernier caliper

2.3. Tasks: Determination of refractive index of polymer slab [6 marks]

1) Derive expressions for the conditions of constructive and destructive interferences in terms of θ_1, t, λ and n.

   [1 mark]

2) Using only the experimental components listed in Section 2.2, set up an experiment to measure the receiver output S as a function of the angle of incidence θ_1 in the range from 40° to 75°. Sketch your experimental setup, clearly showing the angles of incidence and reflection and the position of the film on the rotating table. Mark all components using the labels given on page 2. Tabulate your data. Plot the receiver output S versus the angle of incidence θ_1. Determine accurately the angles corresponding to constructive and destructive interferences.

   [3 marks]

3) Assuming that the refractive index of air is 1.00, determine the order of interference m and the refractive index of the polymer slab n. Write the values of m and n on the answer sheet.

[1.5 marks]

4) Carry out error analysis for your results and estimate the uncertainty of n. Write the value of the uncertainty Δn on the answer sheet.

[0.5 marks]

Note:

- ***The lens should be placed in front of the microwave transmitter with the planar surface facing the transmitter to obtain a quasi-parallel microwave beam. The distance between the planar surface of the lens and the aperture of transmitter horn should be 3 cm.***
- ***For best results, maximize the distance between the transmitter and receiver.***
- ***Deviations of the microwave emitted by transmitter from a plane wave may cause extra peaks in the observed pattern. In the prescribed range from 40^o to 75^o, only one maximum and one minimum exist due to interference.***

Part 3: Frustrated Total Internal Reflection

3.1. Introduction

The phenomenon of total internal reflection (TIR) may occur when the plane wave travels from an optically dense medium to less dense medium. However, instead of TIR at the interface as predicted by geometrical optics, the incoming wave in reality penetrates into the less dense medium and travels for some distance parallel to the interface before being scattered back to the denser medium (see Figure 3.1). This effect can be described by a shift D of the reflected beam, known as the Goos-Hänchen shift.

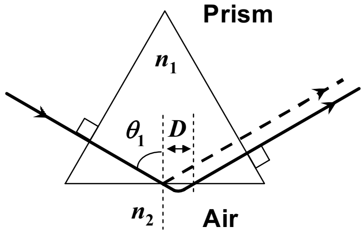


Figure 3.1: A sketch illustrating an EM wave undergoing total internal reflection in a prism. The shift D parallel to the surface in air represents the Goos-Hänchen shift

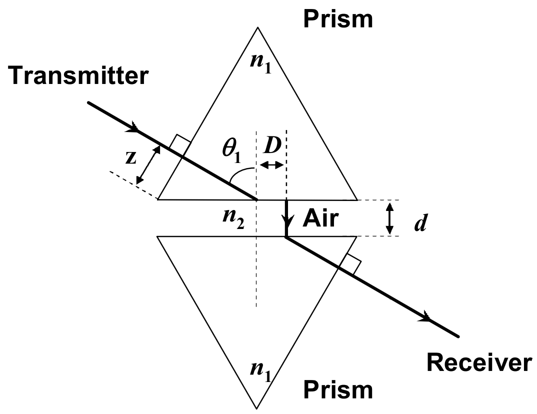


Figure 1.2: A sketch of the experimental setup showing the prisms and the air gap of distance *d*. The shift *D* parallel to the surface in air represents the Goos-Hänchen shift. *z* is the distance from the tip of the prism to the central axis of the transmitter.

If another medium of refractive index n_1 (i.e. made of the same material as the first medium) is placed at a small distance *d* to the first medium as shown in Figure 3.2, tunneling of the EM wave through the second medium occurs. This intriguing phenomenon is known as the *frustrated total internal reflection* (FTIR). The intensity of the transmitted wave, I_t, decreases exponentially with the distance *d*:

$$I_t = I_0 \exp(-2\gamma d) \tag{3.1}$$

where I_0 is the intensity of the incident wave and γ is:

$$\gamma = \frac{2\pi}{\lambda}\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1} \tag{3.2}$$

where λ is the wavelength of EM wave in medium 2 and n_2 is the refractive index of medium 2 (assume that the refractive index of medium 2, air, is 1.00).

3.2. List of components

1) Microwave transmitter Ⓐ with holder Ⓒ

2) Microwave receiver Ⓑ with holder Ⓒ
3) Plano-cylindrical lens Ⓝ with holder Ⓜ
4) 2 equilateral wax prisms Ⓞ with holder Ⓚ and rotating table Ⓛ acting as a holder
5) Digital multimeter Ⓓ
6) Goniometer Ⓙ
7) Ruler

3.3. Description of the Experiment

Using only the list of components described in Section 3.2, set up an experiment to investigate the variation of the intensity I_t as a function of the air gap separation d in FTIR. For consistent results, please take note of the following:

- Use one arm of the goniometer for this experiment.
- Choose the prism surfaces carefully so that they are parallel to each other.
- The distance from the centre of the curved surface of the lens should be 2 cm from the surface of the prism.
- Place the detector such that its horn is in contact with the face of the prism.
- For each value of d, adjust the position of the microwave receiver along the prism surface to obtain the maximum signal.
- Make sure that the digital multi-meter is on the 2mA range. Collect data starting from $d = 0.6$ cm. Discontinue the measurements when the reading of the multimeter falls below 0.20 mA.

3.4. Tasks: Determination of refractive index of prism material [6 marks]

Task 1

Sketch your final experimental setup and mark all components using the labels given at page 2. In your sketch, record the value of the distance z (see Figure 3.2), the distance from the tip of the prism to the central axis of the transmitter.

[1 Mark]

Task 2

Perform your experiment and tabulate your data. Perform this task twice.

[2.1 Marks]

Task 3

(a) By plotting appropriate graphs, determine the refractive index, n_1, of the prism with error analysis.

(b) Write the refractive index n_1, and its uncertainty Δn_1, of the prism in the answer sheet provided.

[2.9 Marks]

Part 4: Microwave diffraction of a metal-rod lattice: Bragg reflection

4.1. Introduction

Bragg's Law

The lattice structure of a real crystal can be examined using Bragg's Law,

$$2d \sin\theta = m\lambda \tag{4.1}$$

where d refers to the distance between a set of parallel crystal planes that "reflect" the X-ray; m is the order of diffraction and θ is the angle between the incident X-ray beam and the crystal planes. Bragg's law is also commonly known as Bragg's reflection or X-ray diffraction.

Metal-rod lattice

Because the wavelength of the X-ray is comparable to the lattice constant of the crystal, traditional Bragg's diffraction experiment is performed using X-ray. For microwave, however, diffraction occurs in lattice structures with much larger lattice constant, which can be measured easily with a ruler.

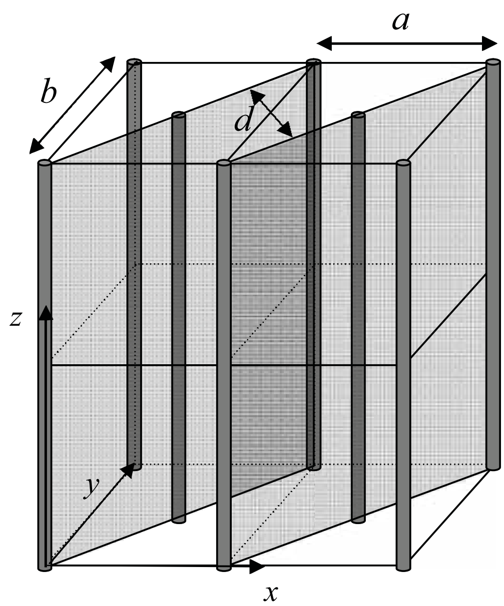


Figure 4.1: A metal-rod lattice of lattice constants *a* and *b*, and interplanar spacing *d*.

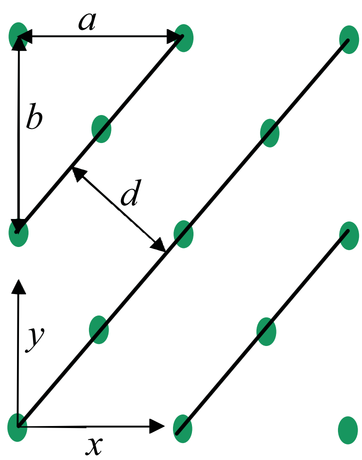


Figure 4.2: Top-view of the metal-rod lattice shown in Fig. 4.1 (not to scale). The lines denote diagonal planes of the lattice.

In this experiment, the Bragg law is used to measure the lattice constant of a lattice made of metal rods. An example of such metal-rod lattice is shown in Fig. 4.1, where the metal rods are shown as thick vertical lines. The lattice planes along the diagonal direction of the *xy*-plane are shown as shaded planes. Fig. 4.2 shows the top-view (looking down along the *z*-axis) of the metal-rod lattice, where the points represent the rods and the lines denote the diagonal lattice planes.

4.2. List of components

1) Microwave transmitter Ⓐ with holder Ⓒ
2) Microwave receiver Ⓑ with holder Ⓒ
3) Plano-cylindrical lens Ⓝ with holder Ⓜ
4) Sealed box containing a metal-rod lattice Ⓘ
5) Rotating table Ⓛ
6) Digital multimeter Ⓓ
7) Goniometer Ⓙ

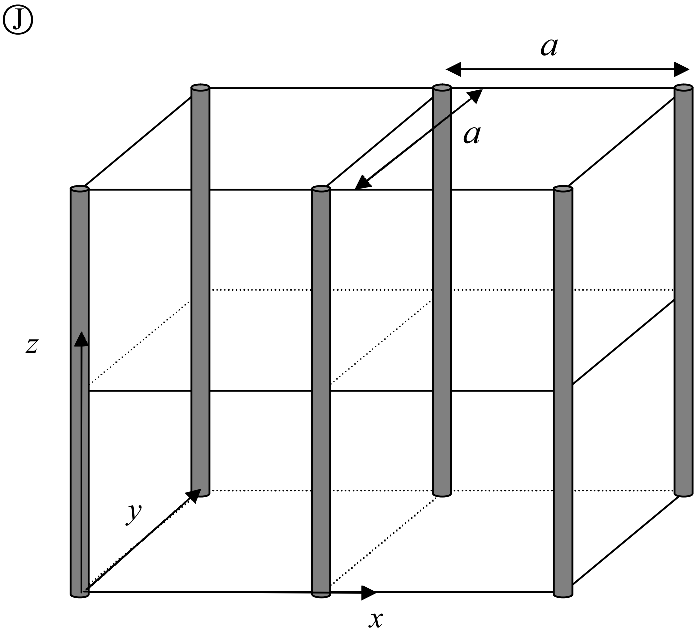


Figure 4.3: A simple square lattice.

In this experiment, you are given a ***simple*** square lattice made of metal rods, as illustrated in Fig. 4.3. The lattice is sealed in a box. You are asked to derive the lattice constant *a* of the lattice from the experiment. DO NOT open the box. No marks will be given to the experimental results if the seal is found broken after the experiment.

4.3. Tasks: Determination of lattice constant of given simple square lattice [6 Marks]

Task 1

Draw a top-view diagram of the simple square lattice shown in Fig. 4.3. In the diagram, indicate the lattice constant a of the given lattice and the interplanar spacing d of the diagonal planes. With the help of this diagram, derive Bragg's Law.

[1 Mark]

Task 2

Using Bragg's law and the apparatus provided, design an experiment to perform Bragg diffraction experiment to determine the lattice constant a of the lattice.

(a) Sketch the experimental set up. Mark all components using the labels in page 2 and indicate clearly the angle between the axis of the transmitter and lattice planes, θ, and the angle between the axis of the transmitter and the axis of the receiver, ζ. In your experiment, measure the diffraction on the diagonal planes the direction of which is indicated by the red line on the box.

[1.5 Marks]

(b) Carry out the diffraction experiment for $20° \leq \theta \leq 50°$. In this range, you will only observe the first order diffraction. In the answer sheet, tabulate your results and record both the θ and ζ.

[1.4 Marks]

(c) Plot the quantity proportional to the intensity of diffracted wave as a function of θ.

[1.3 Marks]

(d) Determine the lattice constant a using the graph and estimate the experimental error.

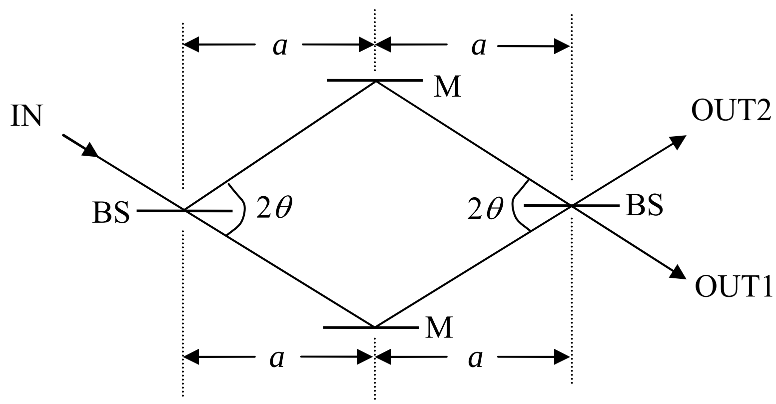
[0.8 Marks]

Note:

1. ***For best results, the transmitter should remain fixed during the experiment. The separation between the transmitter and the lattice, as well as that between lattice and receiver should be about 50 cm.***
2. ***Use only the diagonal planes in this experiment. Your result will not be correct if you try to use any other planes.***
3. ***The face of the lattice box with the red diagonal line must be at the top.***
4. ***To determine the position of the diffraction peak with better accuracy, use a number of data points around the peak position.***

Theory Question 1: Gravity in a Neutron Interferometer

Enter all your answers into the **Answer Script**.



BS - Beam Splitters M - Mirror

Figure 1a

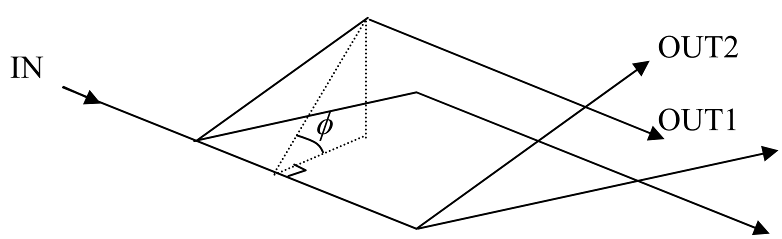


Figure 1b

Physical situation We consider the situation of the famous neutron-interferometer experiment by Collela, Overhauser and Werner, but idealize the set-up inasmuch as we shall assume perfect beam splitters and mirrors within the interferometer. The experiment studies the effect of the gravitational pull on the de Broglie waves of neutrons.

The symbolic representation of this interferometer in analogy to an optical interferometer is shown in Figure 1a. The neutrons enter the interferometer through the IN port and follow the two paths shown. The neutrons are detected at either one of the two output ports, OUT1 or OUT2. The two paths enclose a diamond-shaped area, which is typically a few cm^2 in size.

The neutron de Broglie waves (of typical wavelength of 10^{-10} m) interfere such that all neutrons emerge from the output port OUT1 if the interferometer plane is horizontal. But when the interferometer is tilted around the axis of the incoming neutron beam by angle ϕ (Figure 1b), one observes a ϕ dependent redistribution of the neutrons between the two output ports OUT1 and OUT2.

Geometry For $\phi = 0°$ the interferometer plane is horizontal; for $\phi = 90°$ the plane is vertical with the output ports above the tilt axis.

1.1 (1.0) How large is the diamond-shaped area A enclosed by the two paths of the interferometer?

1.2 (1.0) What is the height H of output port OUT1 above the horizontal plane of the tilt axis?

Express A and H in terms of a, θ, and ϕ.

Optical path length The optical path length N_{opt} (a number) is the ratio of the geometrical path length (a distance) and the wavelength λ. If λ changes along the path, N_{opt} is obtained by integrating λ^{-1} along the path.

1.3 (3.0) What is the difference ΔN_{opt} in the optical path lengths of the two paths when the interferometer has been tilted by angle ϕ? Express your answer in terms of a, θ, and ϕ as well as the neutron mass M, the de Broglie wavelength λ_0 of the incoming neutrons, the gravitational acceleration g, and Planck's constant h.

1.4 (1.0) Introduce the volume parameter

$$V = \frac{h^2}{gM^2}$$

and express ΔN_{opt} solely in terms of A, V, λ_0, and ϕ. State the value of V for $M = 1.675 \times 10^{-27}\,\text{kg}$, $g = 9.800\,\text{m s}^{-2}$, and $h = 6.626 \times 10^{-34}\,\text{J s}$.

1.5 (2.0) How many cycles — from high intensity to low intensity and back to high intensity — are completed by output port OUT1 when ϕ is increased from $\phi = -90°$ to $\phi = 90°$?

Experimental data The interferometer of an actual experiment was characterized by $a = 3.600\,\text{cm}$ and $\theta = 22.10°$, and 19.00 full cycles were observed.

1.6 (1.0) How large was λ_0 in this experiment?

1.7 (1.0) If one observed 30.00 full cycles in another experiment of the same kind that uses neutrons with $\lambda_0 = 0.2000\,\text{nm}$, how large would be the area A?

Hint: If $|\alpha x| << 1$, it is permissible to replace $(1+x)^{\alpha}$ by $1+\alpha x$.

Country Code	Student Code	Question Number
		1

Answer Script

Geometry

For Examiners Use Only

1.1 The area is

$A =$

1.0

1.2 The height is

$H =$

1.0

Country Code	Student Code	Question Number
		1

Optical path length

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1.3 In terms of a, θ, ϕ, M, λ_0, g, and h:

$\Delta N_{\text{opt}} =$

3.0

1.4 In terms of A, V, λ_0, and ϕ:

$\Delta N_{\text{opt}} =$

0.8

The numerical value of V is

$V =$

0.2

1.5 The number of cycles is

# of cycles =

2.0

Country Code	Student Code	Question Number
		1

Experimental data

1.6 The de Broglie wavelength was

$\lambda_0 =$

1.7 The area is

$A =$

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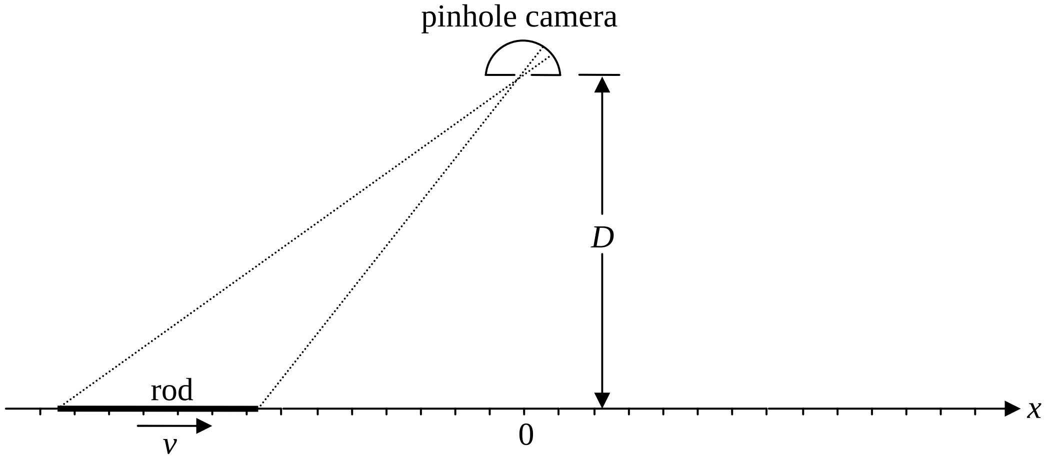
1.0

1.0

Theory Question 2: Watching a Rod in Motion

Enter all your answers into the **Answer Script**.



Physical situation A pinhole camera, with the pinhole at $x = 0$ and at distance D from the x axis, takes pictures of a rod, by opening the pinhole for a very short time. There are equidistant marks along the x axis by which the *apparent length* of the rod, as it is seen on the picture, can be determined from the pictures taken by the pinhole camera. On a picture of the rod *at rest*, its length is L. However, the rod is *not* at rest, but is moving with constant velocity υ along the x axis.

Basic relations A picture taken by the pinhole camera shows a tiny segment of the rod at position $\tilde{x}$.

2.1 (0.6) What is the *actual position* x of this segment at the time when the picture is taken? State your answer in terms of $\tilde{x}$, D, L, υ, and the speed of light c =3.00×10^8 m s^{-1}. Employ the quantities

$$\beta = \frac{\upsilon}{c} \text{ and } \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

if they help to simplify your result.

2.2 (0.9) Find also the corresponding inverse relation, that is: express $\tilde{x}$ in terms of x, D, L, υ, and c.

Note: The *actual position* is the position in the frame in which the camera is at rest

Apparent length of the rod The pinhole camera takes a picture at the instant when the actual position of the center of the rod is at some point x_0.

2.3 (1.5) In terms of the given variables, determine the apparent length of the rod on this picture.

2.4 (1.5) Check one of the boxes in the **Answer Script** to indicate how the apparent length changes with time.

Symmetric picture One pinhole-camera picture shows both ends of the rod at the same distance from the pinhole.

2.5 (0.8) Determine the apparent length of the rod on this picture.

2.6 (1.0) What is the actual position of the middle of the rod at the time when this picture is taken?

2.7 (1.2) Where does the picture show the image of the middle of the rod?

Very early and very late pictures The pinhole camera took one picture very early, when the rod was very far away and approaching, and takes another picture very late, when the rod is very far away and receding. On one of the pictures the apparent length is 1.00 m, on the other picture it is 3.00 m.

2.8 (0.5) Check the box in the **Answer Script** to indicate which length is seen on which picture.

2.9 (1.0) Determine the velocity υ.

2.10 (0.6) Determine the length L of the rod at rest.

2.11 (0.4) Infer the apparent length on the symmetric picture.

Country Code	Student Code	Question Number
		2

Answer Script

For Examiners Use Only

Basic Relations

2.1 x value for given $\tilde{x}$ value:

$x =$

0.6

2.2 $\tilde{x}$ value for given x value:

$\tilde{x} =$

0.9

Apparent length of the rod

2.3 The apparent length is

$\tilde{L}(x_0) =$

1.5

2.4 Check one: The apparent length

- ☐ increases first, reaches a maximum value, then decreases.
- ☐ decreases first, reaches a minimum value, then increases.
- ☐ decreases all the time.
- ☐ increases all the time.

1.5

Country Code	Student Code	Question Number
		2

Symmetric picture

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2.5 The apparent length is **0.8**

$\tilde{L} =$

2.6 The actual position of the middle of the rod is **1.0**

$x_0 =$

2.7 The picture shows the middle of the rod at a distance **1.2**

$l =$

from the image of the front end of the rod.

Country Code	Student Code	Question Number
		2

For Examiners Use Only

Very early and very late pictures

2.8 Check one:

□ The apparent length is 1 m on the early picture and 3 m on the late picture.

□ The apparent length is 3 m on the early picture and 1 m on the late picture.

0.5

2.9 The velocity is

$\upsilon =$

1.0

2.10 The rod has length

$L =$

at rest.

0.6

2.11 The apparent length on the symmetric picture is

$\tilde{L} =$

0.4

Theory Question 3

This question consists of five independent parts. Each of them asks for an estimate of an order of magnitude only, not for a precise answer. Enter all your answers into the **Answer Script**.

Digital Camera Consider a digital camera with a square CCD chip with linear dimension L = 35 mm having N_p = 5 Mpix (1 Mpix = 10^6 pixels). The lens of this camera has a focal length of f = 38 mm. The well known sequence of numbers (2, 2.8, 4, 5.6, 8, 11, 16, 22) that appear on the lens refer to the so called F-number, which is denoted by $F\#$ and defined as the ratio of the focal length and the diameter D of the aperture of the lens, $F\# = f / D$.

3.1 (1.0) Find the best possible spatial resolution $\Delta x_{\min}$, at the chip, of the camera as limited by the lens. Express your result in terms of the wavelength λ and the F-number $F\#$ and give the numerical value for λ = 500 nm.

3.2 (0.5) Find the necessary number N of Mpix that the CCD chip should possess in order to match this optimal resolution.

3.3 (0.5) Sometimes, photographers try to use a camera at the smallest practical aperture. Suppose that we now have a camera of N_0 = 16 Mpix, with the chip size and focal length as given above. Which value is to be chosen for $F\#$ such that the image quality is not limited by the optics?

3.4 (0.5) Knowing that the human eye has an approximate angular resolution of ϕ = 2 arcmin and that a typical photo printer will print a minimum of 300 dpi (dots per inch), at what minimal distance z should you hold the printed page from your eyes so that you do not see the individual dots?

Data 1 inch = 25.4 mm
1 arcmin = 2.91×10^{-4} rad

Hard-boiled egg An egg, taken directly from the fridge at temperature $T_0 = 4°\text{C}$, is dropped into a pot with water that is kept boiling at temperature T_1.

3.5 (0.5) How large is the amount of energy U that is needed to get the egg coagulated?

3.6 (0.5) How large is the heat flow J that is flowing into the egg?

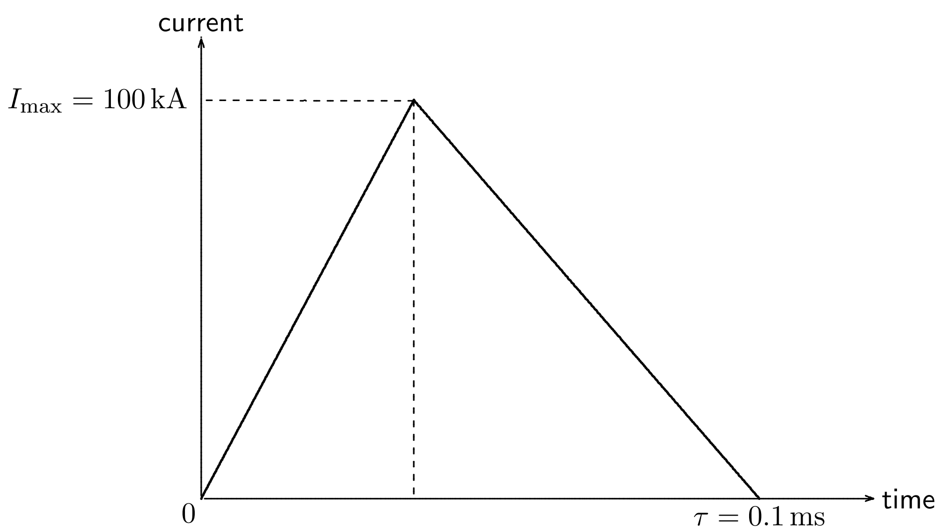
3.7 (0.5) How large is the heat power P transferred to the egg?

3.8 (0.5) For how long do you need to cook the egg so that it is hard-boiled?

Hint You may use the simplified form of Fourier's Law $J = \kappa \Delta T / \Delta r$, where ΔT is the temperature difference associated with Δr, the typical length scale of the problem. The heat flow J is in units of W m^{-2}.

Data Mass density of the egg: $\mu = 10^3\ \text{kg m}^{-3}$
Specific heat capacity of the egg: $C = 4.2\ \text{J K}^{-1}\ \text{g}^{-1}$
Radius of the egg: $R = 2.5\ \text{cm}$
Coagulation temperature of albumen (egg protein): $T_c = 65°\text{C}$
Heat transport coefficient: $\kappa = 0.64\ \text{W K}^{-1}\ \text{m}^{-1}$ (assumed to be the same for liquid and solid albumen)

Lightning An oversimplified model of lightning is presented. Lightning is caused by the build-up of electrostatic charge in clouds. As a consequence, the bottom of the cloud usually gets positively charged and the top gets negatively charged, and the ground below the cloud gets negatively charged. When the corresponding electric field exceeds the breakdown strength value of air, a disruptive discharge occurs: this is lightning.



Idealized current pulse flowing between the cloud and the ground during lightning.

Answer the following questions with the aid of this simplified curve for the current as a function of time and these data:

Distance between the bottom of the cloud and the ground: $h = 1$ km;
Breakdown electric field of humid air: $E_0 = 300$ kV m^{-1};
Total number of lightning striking Earth per year: 32×10^6;
Total human population: 6.5×10^9 people.

3.9 (0.5) What is the total charge Q released by lightning?

3.10 (0.5) What is the average current I flowing between the bottom of the cloud and the ground during lightning?

3.11 (1.0) Imagine that the energy of all storms of one year is collected and equally shared among all people. For how long could you continuously light up a 100 W light bulb for your share?

Capillary Vessels Let us regard blood as an incompressible viscous fluid with mass density μ similar to that of water and dynamic viscosity $\eta = 4.5$ g m^{-1} s^{-1}. We model blood vessels as circular straight pipes with radius r and length L and describe the blood flow by Poiseuille's law,

$$\Delta p = RD,$$

the Fluid Dynamics analog of Ohm's law in Electricity. Here Δp is the pressure difference between the entrance and the exit of the blood vessel, $D = S\upsilon$ is the volume flow through the cross-sectional area S of the blood vessel and υ is the blood velocity. The hydraulic resistance R is given by

$$R = \frac{8\eta L}{\pi r^4}.$$

For the systemic blood circulation (the one flowing from the left ventricle to the right auricle of the heart), the blood flow is $D \approx 100$ cm^{3}s^{-1} for a man at rest. Answer the following questions under the assumption that all capillary vessels are connected in parallel and that each of them has radius $r = 4$ µm and length $L = 1$ mm and operates under a pressure difference $\Delta p = 1$ kPa.

3.12 (1.0) How many capillary vessels are in the human body?

3.13 (0.5) How large is the velocity υ with which blood is flowing through a capillary vessel?

Skyscraper At the bottom of a 1000 m high skyscraper, the outside temperature is T_{bot}= 30°C. The objective is to estimate the outside temperature T_{top} at the top. Consider a thin slab of air (ideal nitrogen gas with adiabatic coefficient γ = 7/5) rising slowly to height z where the pressure is lower, and assume that this slab expands adiabatically so that its temperature drops to the temperature of the surrounding air.

3.14 (0.5) How is the fractional change in temperature dT/T related to dp/p, the fractional change in pressure?

3.15 (0.5) Express the pressure difference dp in terms of dz, the change in height.

3.16 (1.0) What is the resulting temperature at the top of the building?

<u>Data</u> Boltzmann constant: $k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Mass of a nitrogen molecule: $m = 4.65 \times 10^{-26}\,\mathrm{kg}$
Gravitational acceleration: $g = 9.80\,\mathrm{m\,s^{-2}}$

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Answer Script

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Digital Camera

3.1 The best spatial resolution is

(formula:) $\Delta x_{\min} =$

which gives
(numerical:) $\Delta x_{\min} =$

for $\lambda = 500\,\text{nm}$.

0.7

0.3

3.2 The number of Mpix is

$N =$

0.5

3.3 The best F-number value is

$F\# =$

0.5

3.4 The minimal distance is

$z =$

0.5

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Hard-boiled egg

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3.5 The required energy is

$U =$

0.5

3.6 The heat flow is

$J =$

0.5

3.7 The heat power transferred is

$P =$

0.5

3.8 The time needed to hard-boil the egg is

$\tau =$

0.5

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Lightning

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3.9 The total charge is **0.5**

$Q =$

3.10 The average current is **0.5**

$I =$

3.11 The light bulb would burn for the duration **1.0**

$t =$

Capillary Vessels

3.12 There are **1.0**

$N =$

capillary vessels in a human body.

3.13 The blood flows with velocity **0.5**

$\upsilon =$

Country Code	Student Code	Question Number
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Skyscraper

3.14 The fractional change in temperature is

$$\frac{dT}{T} =$$

3.15 The pressure difference is

$$dp =$$

3.16 The temperature at the top is

$$T_{\text{top}} =$$

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0.5

0.5

1.0