In physics, whenever we have an equality relation, both sides of the equation should be of the same type i.e. they must have the same dimensions. For example you cannot have a situation where the quantity on the right-hand side of the equation represents a length and the quantity on the left-hand side represents a time interval. Using this fact, sometimes one can nearly deduce the form of a physical relation without solving the problem analytically. For example if we were asked to find the time it takes for an object to fall from a height of h under the influence of a constant gravitational acceleration g, we could argue that one only needs to build a quantity representing a time interval, using the quantities g and h and the only possible way of doing this is $T = a(h/g)^{1/2}$. Notice that this solution includes an as yet undetermined coefficient a which is *dimensionless* and thus cannot be determined, using this method. This coefficient can be a number such as 1, $1/2$, $\sqrt{3}$, π, or any other real number. This method of deducing physical relations is called *dimensional analysis*. In dimensional analysis the dimensionless coefficients are not important and we do not need to write them. Fortunately in most physical problems these coefficients are of the order of 1 and eliminating them does not change the order of magnitude of the physical quantities. Therefore, by applying the dimensional analysis to the above problem, one obtains $T = (h/g)^{1/2}$.

Generally, the dimensions of a physical quantity are written in terms of the dimensions of four fundamental quantities: M (mass), L (length), T (time), and K (temperature). The dimensions of an arbitrary quantity, x is denoted by $[x]$. As an example, to express the dimensions of velocity v, kinetic energy E_k, and heat capacity C_V we write: $[v] = LT^{-1}$, $[E_k] = ML^2T^{-2}$, $[C_V] = ML^2T^{-2}K^{-1}$.

1 Fundamental Constants and Dimensional Analysis

1.1	Find the dimensions of *the fundamental constants,* i.e. the Planck's constant, h, the speed of light, c, the universal constant of gravitation, G, and the Boltzmann constant, k_B, in terms of the dimensions of length, mass, time, and temperature.	0.8

The Stefan-Boltzmann law states that the black body emissive power which is the total energy radiated per unit surface area of a black body in unit time is equal to $\sigma\theta^4$ where σ is the Stefan-Boltzmann's constant and θ is the absolute temperature of the black body.

1.2	Determine the dimensions of the Stefan-Boltzmann's constant in terms of the dimensions of length, mass, time, and temperature.	0.5

The Stefan-Boltzmann's constant is not a fundamental constant and one can write it in terms of fundamental constants i.e. one can write $\sigma = a\, h^{\alpha} c^{\beta} G^{\gamma} k_B^{\ \delta}$. In this relation a is a dimensionless parameter of the order of 1. As mentioned before, the exact value of a is not significant from our viewpoint, so we will set it equal to 1.

1.3	Find α, β, γ, and δ using dimensional analysis.	1.0

2 Physics of Black Holes

In this part of the problem, we would like to find out some properties of black holes using dimensional analysis. According to a certain theorem in physics known as the *no hair theorem*, all the characteristics of the black hole which we are considering in this problem depend only on the mass of the black hole. One characteristic of a black hole is the area of its *event horizon*. Roughly speaking, the event horizon is the boundary of the black hole. Inside this boundary, the gravity is so strong that even light cannot emerge from the region enclosed by the boundary.

We would like to find a relation between the mass of a black hole, m, and the area of its event horizon, A. This area depends on the mass of the black hole, the speed of light, and the universal constant of gravitation. As in 1.3 we shall write $A = G^{\alpha} c^{\beta} m^{\gamma}$.

2.1	Use dimensional analysis to find α, β, and γ.	0.8

From the result of 2.1 it becomes clear that the area of the event horizon of a black hole increases with its mass. From a classical point of view, nothing comes out of a black hole and therefore in all physical processes the area of the event horizon can only increase. In analogy with the second law of thermodynamics, Bekenstein proposed to assign entropy, S, to a black hole, proportional to the area of its event horizon i.e. $S = \eta A$. This conjecture has been made more plausible using other arguments.

2.2	Use the thermodynamic definition of entropy $dS = dQ/\theta$ to find the dimensions of entropy. dQ is the exchanged heat and θ is the absolute temperature of the system.	0.2

2.3	As in 1.3, express the dimensioned constant η as a function of the fundamental constants h, c, G, and k_B.	1.1

*Do **not** use dimensional analysis for the rest of problem, but you may use the results you have obtained in previous sections.*

3 Hawking Radiation

With a semi-quantum mechanical approach, Hawking argued that contrary to the classical point of view, black holes emit radiation similar to the radiation of a black body at a temperature which is called the *Hawking temperature*.

3.1	Use $E = mc^2$, which gives the energy of the black hole in terms of its mass, and the laws of thermodynamics to express the Hawking temperature θ_H of a black hole in terms of its mass and the fundamental constants. Assume that the black hole does no work on its surroundings.	0.8

3.2	The mass of an isolated black hole will thus change because of the Hawking radiation. Use Stefan-Boltzmann's law to find the dependence of this rate of change on the Hawking temperature of the black hole, θ_H and express it in terms of mass of the black hole and the fundamental constants.	0.7

3.3	Find the time t^*, that it takes an isolated black hole of mass m to evaporate completely i.e. to lose all its mass.	1.1

From the viewpoint of thermodynamics, black holes exhibit certain exotic behaviors. For example the heat capacity of a black hole is negative.

3.4	Find the heat capacity of a black hole of mass m.	0.6

4 Black Holes and the Cosmic Background Radiation

Consider a black hole exposed to the cosmic background radiation. The cosmic background radiation is a black body radiation with a temperature θ_B which fills the entire universe. An object with a total area A will thus receive an energy equal to $\sigma \theta_B^{\ 4} \times A$ per unit time. A black hole, therefore, loses energy through Hawking radiation and gains energy from the cosmic background radiation.

4.1	Find the rate of change of a black hole's mass, in terms of the mass of the black hole, the temperature of the cosmic background radiation, and the fundamental constants.	0.8
4.2	At a certain mass, m^*, this rate of change will vanish. Find m^* and express it in terms of θ_B and the fundamental constants.	0.4
4.3	Use your answer to 4.2 to substitute for θ_B in your answer to part 4.1 and express the rate of change of the mass of a black hole in terms of m, m^*, and the fundamental constants.	0.2
4.4	Find the Hawking temperature of a black hole at thermal equilibrium with cosmic background radiation.	0.4
4.5	Is the equilibrium stable or unstable? Why? (Express your answer mathematically)	0.6

Experimental Problem

Determination of energy band gap of semiconductor thin films

I. Introduction

Semiconductors can be roughly characterized as materials whose electronic properties fall somewhere between those of conductors and insulators. To understand semiconductor electronic properties, one can start with the *photoelectric effect* as a well-known phenomenon. The photoelectric effect is a quantum electronic phenomenon, in which photoelectrons are emitted from the matter through the absorption of sufficient energy from electromagnetic radiation (i.e. photons). The minimum energy which is required for the emission of an electron from a metal by light irradiation (*photoelectron*) is defined as "*work function"*. Thus, only photons with a frequency ν higher than a characteristic threshold, i.e. with an energy $h\nu$ (h is the Planck's constant) more than the material's work function, are able to knock out the photoelectrons.

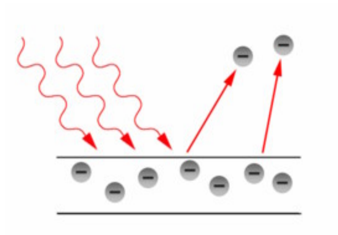

Figure 1. An illustration of photoelectron emission from a metal plate: The incoming photon should have an energy which is more than the work function of the material.

In fact, the concept of work function in the photoelectric process is similar to the concept of the energy band gap of a semiconducting material. In solid state physics, the band gap E_g is the energy difference between the top of the valence band and the bottom of the conduction band of insulators and semiconductors. The valence band is completely filled with electrons, while the conduction band is empty however electrons can go from the valence band to the conduction band if they acquire sufficient energy (at least equal to the band gap energy).The semiconductor's conductivity strongly depends on its energy band gap.

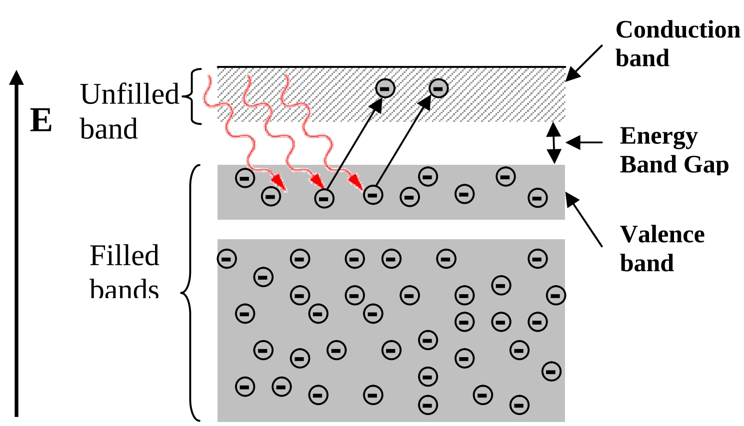

Figure 2. Energy band scheme for a semiconductor.

Band gap engineering is the process of controlling or altering the band gap of a material by controlling the composition of certain semiconductor alloys. Recently, it has been shown that by changing the nanostructure of a semiconductor it is possible to manipulate its band gap.

In this experiment, we are going to obtain the energy band gap of a thin-film semiconductor containing nano-particle chains of iron oxide (Fe_2O_3) by using an optical method. To measure the band gap, we study the optical absorption properties of the transparent film using its optical transmission spectrum. As a rough statement, the absorption spectra shows a sharp increase when the energy of the incident photons equals to the energy band gap.

II. Experimental Setup

You will find the following items on your desk:

1. A large white box containing a spectrometer with a halogen lamp.
2. A small box containing a sample, a glass substrate, a sample-holder, a grating, and a photoresistor.
3. A multimeter.
4. A calculator.
5. A ruler.
6. A card with a hole punched in its center.
7. A set of blank labels.

The spectrometer contains a goniometer with a precision of $5'$. The Halogen lamp acts as the source of radiation and is installed onto the fixed arm of the spectrometer (for detailed information see the enclosed "Description of Apparatus").

The small box contains the following items:

1. A sample-holder with two windows: a glass substrate coated with Fe_2O_3 film mounted on one window and an uncoated glass substrate mounted on the other.
2. A photoresistor mounted on its holder, which acts as a light detector.
3. A transparent diffraction grating (600 line/mm).

Note: *Avoid touching the surface of any component in the small box!*

A schematic diagram of the setup is shown in Figure 3:

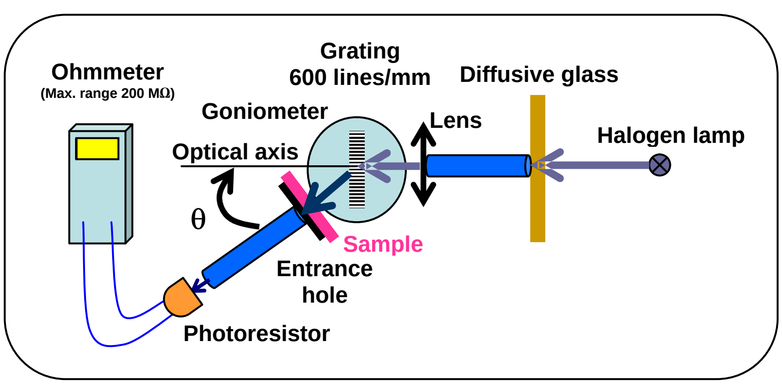

Figure 3. Schematic diagram of the experimental setup.

III. Methods

To obtain the transmission of a film at each wavelength, $T_{film}(\lambda)$, one can use the following formula:

$$T_{film}(\lambda) = I_{film}(\lambda) / I_{glass}(\lambda) \qquad (1)$$

where I_{film} and I_{glass} are respectively the intensity of the light transmitted from the coated glass substrate, and the intensity of the light transmitted from the uncoated glass slide. The value of I can be measured using a light detector such as a photoresistor. In a photoresistor, the electrical resistance decreases when the intensity of the incident light increases. Here, the value of I can be determined from the following relation:

$$I(\lambda) = C(\lambda) R^{-1} \qquad \textbf{(2)}$$

where R is the electrical resistance of the photoresistor, C is a λ-dependent coefficient.

The transparent grating on the spectrometer diffracts different wavelengths of light into different angles. Therefore, to study the variations of T as a function of λ, it is enough to change the angle of the photoresistor (θ') with respect to the optical axis (defined as the direction of the incident light beam on the grating), as shown in Figure 4.

From the principal equation of a diffraction grating:

$$n\lambda = d[\sin(\theta' - \theta_0) + \sin\theta_0] \quad (3)$$

one can obtain the angle θ' corresponding to a particular λ: n is an integer number representing the order of diffraction, d is the period of the grating, and θ_o is the angle the normal vector to the surface of grating makes with the optical axis (see Fig. 4). (In this experiment we shall try to place the grating perpendicular to the optical axis making $\theta_o = 0$, but since this cannot be achieved with perfect precision the error associated with this adjustment will be measured in task 1-e.)

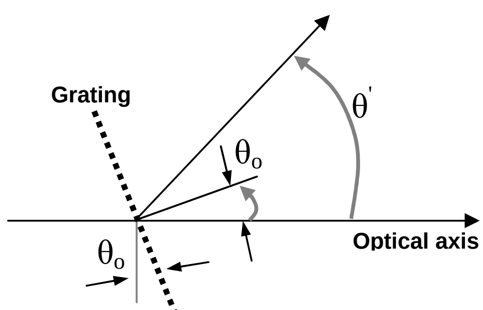

Figure 4. Definition of the angles involved in Equation 3.

Experimentally it has been shown that for photon energies slightly larger than the band gap energy, the following relation holds:

$$\alpha h\nu = A(h\nu - E_g)^{\eta} \quad (4)$$

where α is the absorption coefficient of the film, A is a constant that depends on the film's material, and η is the constant determined by the absorption mechanism of the film's material and structure. Transmission is related to the value of α through the well-known absorption relation:

$$T_{film} = \exp(-\alpha t) \qquad (5)$$

where t is thickness of the film.

IV. Tasks:

0. *Your apparatus and sample box (small box containing the sample holder) are marked with numbers. Write down the* ***Apparatus number*** *and* ***Sample number*** *in their appropriate boxes, in the answer sheet.*

1. <u>Adjustments and Measurements:</u>

1-a	Check the vernier scale and report the maximum precision ($\Delta\theta$).	*0.1 pt*

Note: Magnifying glasses are available on request.

Step1:
To start the experiment, turn on the Halogen lamp to warm up. It would be better not to turn off the lamp during the experiment. Since the halogen lamp heats up during the experiment, please be careful not to touch it.

Place the lamp as far from the lens as possible, this will give you a parallel light beam.

We are going to make a rough zero-adjustment of the goniometer without utilizing the photoresistor. Unlock the rotatable arm with screw 18 (underneath the arm), and visually align the rotatable arm with the optical axis. Now, firmly lock the rotatable arm with screw 18. Unlock the vernier with screw 9 and rotate the stage to 0 on the vernier scale. Now firmly lock the vernier with screw 9 and use the vernier fine-adjustment screw (screw 10) to set the zero of the vernier scale. Place the grating inside its holder. Rotate the grating's stage until the diffraction grating is roughly perpendicular to the optical axis. Place the card with a hole in front of the light source and position the hole such that a beam of light is incident on the grating. Carefully rotate the grating so that the spot of reflected light falls onto the hole. Then the reflected light beam coincides with the incident beam. Now lock the grating's stage by tightening screw 12.

1-b	By measuring the distance between the hole and the grating, estimate the precision of this adjustment ($\Delta\theta_o$).	*0.3 pt*
	Now, by rotating the rotatable arm, determine and report the range of angles for which the first-order diffraction of visible light (from blue to red) is observed.	*0.2 pt*

Step 2:
Now, install the photoresistor at the end of the rotatable arm. To align the system optically, by using the photoresistor, loosen the screw 18, and slightly turn the rotatable arm so that the photoresistor shows a minimum resistance. For fine positioning, firmly lock screw 18, and use the fine adjustment screw of the rotatable arm.

Use the vernier fine-adjustment screw to set the zero of the vernier scale.

1-c	Report the measured minimum resistance value ($R_{\min}^{(0)}$).	*0.1 pt*
	Your zero-adjustment is more accurate now, report the precision of this new adjustment ($\Delta\varphi_o$) . Note: $\Delta\varphi_o$ is the error in this alignment i.e. it is a measure of misalignment of the rotatable arm and the optical axis.	*0.1 pt*

- **Hint:** After this task you should tighten the fixing screws of the vernier. Moreover, tighten the screw of the photoresistor holder to fix it and do not remove it during the experiment.

Step 3:
Move the rotatable arm to the region of the first-order diffraction. Find the angle at which the resistance of the photoresistor is minimum (maximum light intensity). Using the balancing screws, you can slightly change the *tilt* of the grating's stage, to achieve an even lower resistance value.

1-c	Report the minimum value of the observed resistance ($R_{\min}^{(1)}$) in its appropriate box.	*0.1 pt*

It is now necessary to check the perpendicularity of the grating for zero adjustment, again. For this you must use the reflection-coincidence method of Step 1.

Important: From here onwards carry out the experiment in dark (close the cover).

Measurements: Screw the sample-holder onto the rotatable arm. Before you start the measurements, examine the appearance of your semiconductor film (sample). Place the sample in front of the entrance hole S_1 on the rotatable arm such that a uniformly coated part of the sample covers the hole. To make sure that every time you will be working with the same part of the sample make proper markings on the sample holder and the rotatable arm with blank labels.

Attention: At higher resistance measurements it is necessary to allow the photoresistor to relax, therefore for each measurement in this range wait 3 to 4 minutes before recording your measurement.

1-d	Measure the resistance of the photoresistor for the uncoated glass substrate and the glass substrate coated with semiconductor layer as a function of the angle θ (the value read by the goniometer for the angle between the photoresistor and your specified optical axis). Then fill in Table 1d. Note that you need at least 20 data points in the range you found in Step 1b. Carry out your measurement using the appropriate range of your ohmmeter.	*2.0 pt*
	Consider the error associated with each data point. Base your	*1.0 pt*

	answer only on your direct readings of the ohmmeter.	

Step 4:
The precision obtained so far is still limited since it is impossible to align the rotatable arm with the optical axis and/or position the grating perpendicular to the optical axis with 100% precision. So we still need to find the asymmetry of the measured transmission at both sides of the optical axis (resulting from the deviation of the normal to the grating surface from the optical axis (θ_o)).

To measure this asymmetry, follow these steps:

1-e	First, measure T_{film} at $\theta = -20°$. Then, obtain values for T_{film} at some other angles around $+20°$. Complete Table 1e (you can use the values obtained in Table 1d).	*0.6 pt*
	Draw T_{film} versus θ and visually draw a curve.	*0.6 pt*

On your curve find the angle γ for which the value of T_{film} is equal to the T_{film} that you measured at $\theta = -20^o$ ($\gamma \equiv \theta|_{T_{film} = T_{film}(-20°)}$). Denote the difference of this angle with $+20°$ as δ, in other words:

$$\delta = \gamma - 20° \qquad (6)$$

1-e	Report the value of δ in the specified box.	*0.2 pt*

Then for the first-order diffraction, Eq. (3) can be simplified as follows:

$$\lambda = d\sin(\theta - \delta/2), \qquad (7)$$

where θ is the angle read on the goniometer.

2. <u>Calculations:</u>

2-a	Use Eq. (7) to express $\Delta\lambda$ in terms of the errors of the other parameters (assume d is exact and there is no error is associated with it). Also using Eqs. (1), (2), and (5), express ΔT_{film} in terms of R and ΔR.	*0.6 pt*

2-b	Report the range of values of $\Delta\lambda$ over the region of first-order diffraction.	*0.3 pt*

2-c	Based on the measured parameters in Task 1, complete Table 2c for each θ. Note that the wavelength should be calculated using Eq. (7).	*2.4 pt*

2-d	Plot R^{-1}_{glass} and R^{-1}_{film} as a function of wavelength together on the same diagram. Note that on the basis of Eq. (2) behaviors of R^{-1}_{glass} and R^{-1}_{film} can reasonably give us an indication of the way I_{glass} and I_{film} behave, respectively.	*1.5 pt*
	In Table 2d, report the wavelengths at which R_{glass} and R_{film} attain their minimum values.	*0.4 pt*

2-e	For the semiconductor layer (sample) plot T_{film} as a function of wavelength. This quantity also represents the variation of the film transmission in terms of wavelength.	*1.0 pt*

3. <u>Data analysis:</u>

By substituting $\eta = 1/2$ and $A = 0.071$ ((eV)$^{1/2}$/nm) in Eq. (4) one can find values for E_g and t in units of eV and nm, respectively. This will be accomplished by plotting a suitable diagram in an $x-y$ coordinate system and doing an extrapolation in the region satisfying this equation.

3-a	By assuming $x = h\nu$ and $y = (\alpha t h\nu)^2$ and by using your measurements in Task 1, fill in Table 3a for wavelengths around 530 nm and higher. Express your results (x and y) with the correct number of significant figures (digits), based on the estimation of the error on one single data point. <u>Note that $h\nu$ should be calculated in units of eV and wavelength in units of nm</u>. Write the unit of each variable between the parentheses in the top row of the table.	*2.4 pt*

3-b	Plot y versus x.	*2.6 pt*
	Note that the y parameter corresponds to the absorption of the film. Fit a line to the points in the linear region around 530 nm.	
	Specify the region where Eq. (4) is satisfied, by reporting the values of the smallest and the largest x-coordinates for the data points to which you fit the line.	

3-c	Call the slope of this line m, and find an expression for the film thickness (t) and its error (Δt) in terms of m and A (consider A to have no error).	*0.5 pt*

3-d	Obtain the values of E_g and t and their associated errors in units of eV and nm, respectively. Fill in Table 3d.	*3.0 pt*

❖ Some useful physical constants required for your analysis:

- Speed of the light: $c = 3.00\times10^{8}$ m/s
- Plank's constant: $h = 6.63\times10^{-34}$ J.s
- Electron charge: $e = 1.60\times10^{-19}$ C

In this problem we deal with a simplified model of accelerometers designed to activate the safety air bags of automobiles during a collision. We would like to build an electromechanical system in such a way that when the acceleration exceeds a certain limit, one of the electrical parameters of the system such as the voltage at a certain point of the circuit will exceed a threshold and the air bag will be activated as a result.

Note: Ignore gravity in this problem.

1 Consider a capacitor with parallel plates as in Figure 1. The area of each plate in the capacitor is A and the distance between the two plates is d. The distance between the two plates is much smaller than the dimensions of the plates. One of these plates is in contact with a wall through a spring with a spring constant k, and the other plate is fixed. When the distance between the plates is d the spring is neither compressed nor stretched, in other words no force is exerted on the spring in this state. Assume that the permittivity of the air between the plates is that of free vacuum ε_0. The capacitance corresponding to this distance between the plates of the capacitor is $C_0 = \varepsilon_0 A/d$. We put charges $+Q$ and $-Q$ on the plates and let the system achieve mechanical equilibrium.

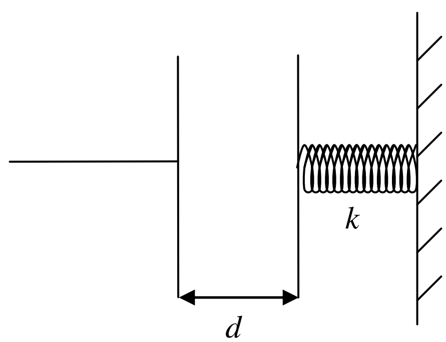

Figure 1

1.1	Calculate the electrical force, F_E, exerted by the plates on each other.	0.8
1.2	Let x be the displacement of the plate connected to the spring. Find x.	0.6
1.3	In this state, what is the electrical potential difference V between the plates of the capacitor in terms of Q, A, d, k?	0.4
1.4	Let C be the capacitance of the capacitor, defined as the ratio of charge to potential difference. Find C/C_0 as a function of Q, A, d and k.	0.3
1.5	What is the total energy, U, stored in the system in terms of Q, A, d and k?	0.6

Figure 2, shows a mass M which is attached to a conducting plate with negligible mass and also to two springs having identical spring constants k. The conducting plate can move back and forth in the space between two fixed conducting plates. All these plates are similar and have the same area A. Thus these three plates constitute two capacitors. As shown in Figure 2, the fixed plates are connected to the given potentials V and $-V$, and the middle plate is connected through a two-state switch to the ground. The wire connected to the movable plate does not disturb the motion of the plate and the three plates will always remain parallel. When the whole complex is not being accelerated, the distance from each fixed plate to the movable plate is d which is much smaller than the dimensions of the plates. The thickness of the movable plate can be ignored.

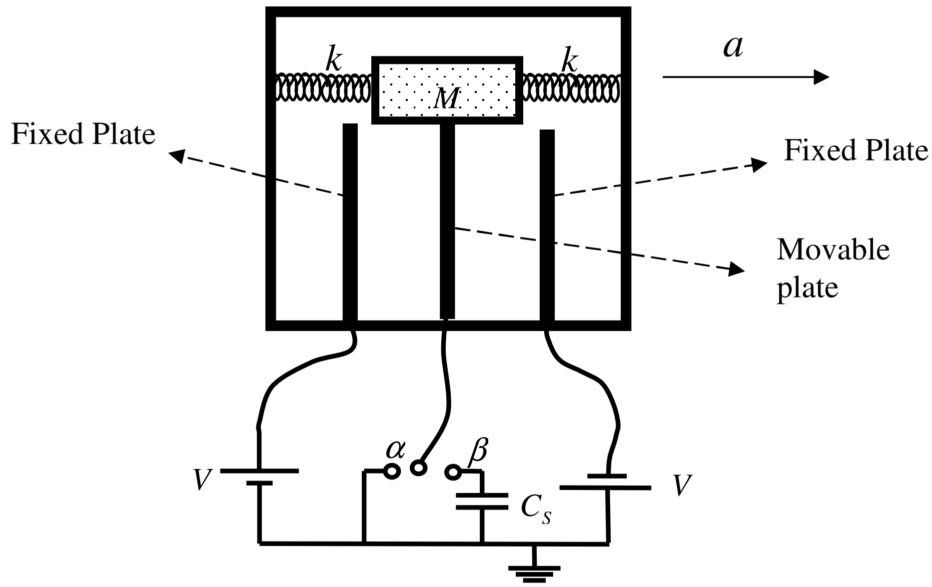

Figure 2

The switch can be in either one of the two states α and β. Assume that the capacitor complex is being accelerated along with the automobile, and the acceleration is constant. Assume that during this constant acceleration the spring does not oscillate and all components of this complex capacitor are in their equilibrium positions, i.e. they do not move with respect to each other, and hence with respect to the automobile.
Due to the acceleration, the movable plate will be displaced a certain amount x from the middle of the two fixed plates.

2 Consider the case where the switch is in state α i.e. the movable plate is connected to the ground through a wire, then

2.1	Find the charge on each capacitor as a function of x.	0.4

2.2	Find the net electrical force on the movable plate, F_E, as a function of x.	0.4

2.3	Assume $d >> x$ and terms of order x^2 can be ignored compared to terms of order d^2. Simplify the answer to the previous part.	0.2

2.4	Write the total force on the movable plate (the sum of the electrical and the spring forces) as $-k_{eff}\, x$ and give the form of k_{eff}.	0.7

2.5	Express the constant acceleration a as a function of x.	0.4

3 Now assume that the switch is in state β i.e. the movable plate is connected to the ground through a capacitor, the capacitance of which is C_S (there is no initial charge on the capacitors). If the movable plate is displaced by an amount x from its central position,

3.1	Find V_S the electrical potential difference across the capacitor C_S as a function of x.	1.5

3.2	Again assume that $d >> x$ and ignore terms of order x^2 compared to terms of order d^2. Simplify your answer to the previous part.	0.2

4 We would like to adjust the parameters in the problem such that the air bag will not be activated in normal braking but opens fast enough during a collision to prevent the driver's head from colliding with the windshield or the steering wheel. As you have seen in Part 2, the force exerted on the movable plate by the springs and the electrical charges can be represented as that of a spring with an effective spring constant k_{eff}. The whole capacitor complex is similar to a *mass and spring* system of mass M and spring constant k_{eff} under the influence of a constant acceleration a, which in this problem is the acceleration of the automobile.

Note: In this part of the problem, the assumption that the mass and spring are in equilibrium under a constant acceleration and hence are fixed relative to the automobile, no longer holds.

Ignore friction and consider the following numerical values for the parameters of the problem:

$$d = 1.0\,\text{cm}, \quad A = 2.5\times10^{-2}\ \text{m}^2, \quad k = 4.2\times10^{3}\ \text{N/m}, \quad \varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2,$$
$$V = 12\,\text{V}, \quad M = 0.15\,\text{kg}.$$

4.1	Using this data, find the ratio of the electrical force you calculated in section 2.3 to the force of the springs and show that one can ignore the electrical forces compared to the spring forces.	0.6

Although we did not calculate the electrical forces for the case when the switch is in the state β, it can be shown that in this situation, quite similarly, the electrical forces are as small and can be ignored.

4.2	If the automobile while traveling with a constant velocity, suddenly brakes with a constant acceleration a, what is the maximum displacement of the movable plate? Give your answer in parameter.	0.6

Assume that the switch is in state β and the system has been designed such that when the electrical voltage across the capacitor reaches $V_S = 0.15\text{V}$, the air bag is activated. We would like the air bag not to be activated during normal braking when the automobile's acceleration is less than the acceleration of gravity $g = 9.8\, m/s^2$, but be activated otherwise.

4.3	How much should C_S be for this purpose?	0.6

We would like to find out if the air bag will be activated fast enough to prevent the driver's head from hitting the windshield or the steering wheel. Assume that as a result of collision, the automobile experiences a deceleration equal to g but the driver's head keeps moving at a constant speed.

4.4	By estimating the distance between the driver's head and the steering wheel, find the time t_1 it takes before the driver's head hits the steering wheel.	0.8

4.5	Find the time t_2 before the air bag is activated and compare it to t_1. Is the air bag activated in time? Assume that airbag opens instantaneously.	0.9

Two stars rotating around their center of mass form a binary star system. Almost half of the stars in our galaxy are binary star systems. It is not easy to realize the binary nature of most of these star systems from Earth, since the distance between the two stars is much less than their distance from us and thus the stars cannot be resolved with telescopes. Therefore, we have to use either photometry or spectrometry to observe the variations in the intensity or the spectrum of a particular star to find out whether it is a binary system or not.

Photometry of Binary Stars

If we are exactly on the plane of motion of the two stars, then one star will occult (pass in front of) the other star at certain times and the intensity of the whole system will vary with time from our observation point. These binary systems are called ecliptic binaries.

1 Assume that two stars are moving on circular orbits around their common center of mass with a constant angular speed ω and we are exactly on the plane of motion of the binary system. Also assume that the surface temperatures of the stars are T_1 and T_2 $(T_1 > T_2)$, and the corresponding radii are R_1 and R_2 $(R_1 > R_2)$, respectively. The total intensity of light, measured on Earth, is plotted in Figure 1 as a function of time. Careful measurements indicate that the intensities of the incident light from the stars corresponding to the minima are respectively 90 and 63 percent of the maximum intensity, I_0, received from both stars $\left(I_0 = 4.8\times10^{-9}\ \text{W/m}^2\right)$. The vertical axis in Figure 1 shows the ratio I/I_0 and the horizontal axis is marked in days.

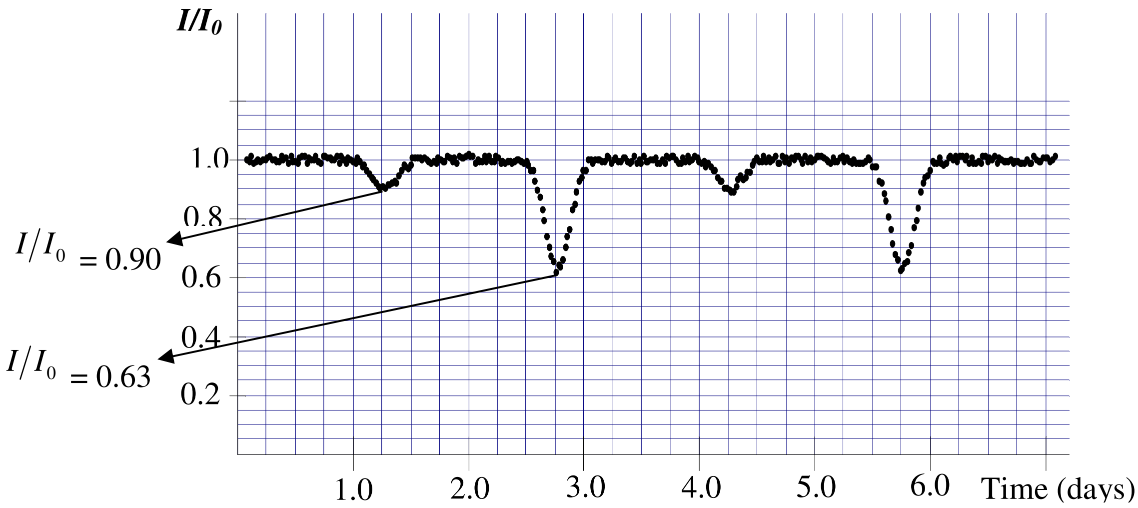

Figure 1. The relative intensity received from the binary star system as a function of time. The vertical axis has been scaled by $I_0 = 4.8\times10^{-9}\ \text{W/m}^2$. Time is given in days.

1.1	Find the period of the orbital motion. Give your answer in **seconds** up to two significant digits. What is the angular frequency of the system in **rad/sec**?	0.8

To a good approximation, the receiving radiation from a star is a uniform black body radiation from a flat disc with a radius equal to the radius of the star. Therefore, the power received from the star is proportional to AT^4 where A is area of the disc and T is the surface temperature of the star.

1.2	Use the diagram in Figure 1 to find the ratios T_1/T_2 and R_1/R_2.	1.6

Spectrometry of Binary Systems

In this section, we are going to calculate the astronomical properties of a binary star by using experimental spectrometric data of the binary system.

Atoms absorb or emit radiation at their certain characteristic wavelengths. Consequently, the observed spectrum of a star contains *absorption lines* due to the atoms in the star's atmosphere. Sodium has a characteristic yellow line spectrum (D_1 line) with a wavelength 5895.9Å (10 Å = 1 nm). We examine the absorption spectrum of atomic Sodium at this wavelength for the binary system of the previous section. The spectrum of the light that we receive from the binary star is Doppler-shifted, because the stars are moving with respect to us. Each star has a different speed. Accordingly the absorption wavelength for each star will be shifted by a different amount. Highly accurate wavelength measurements are required to observe the Doppler shift since the speed of the stars is much less than the speed of light. The speed of the center of mass of the binary system we consider in this problem is much smaller than the orbital velocities of the stars. Hence all the Doppler shifts can be attributed to the orbital velocity of the stars. Table 1 shows the measured spectrum of the stars in the binary system we have observed.

Table 1: Absorption spectrum of the binary star system for the Sodium D_1 line

t/days	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4
λ_1 (Å)	5897.5	5897.7	5897.2	5896.2	5895.1	5894.3	5894.1	5894.6
λ_2 (Å)	5893.1	5892.8	5893.7	5896.2	5897.3	5898.7	5899.0	5898.1

t/days	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8
λ_1 (Å)	5895.6	5896.7	5897.3	5897.7	5897.2	5896.2	5895.0	5894.3
λ_2 (Å)	5896.4	5894.5	5893.1	5892.8	5893.7	5896.2	5897.4	5898.7

(*Note*: There is no need to make a graph of the data in this table)

2 Using Table 1,

2.1	Let v_1 and v_2 be the orbital velocity of each star. Find v_1 and v_2. The speed of light $c = 3.0\times10^8$ m/s. Ignore all relativistic effects.	1.8

2.2	Find the mass ratio of the stars (m_1/m_2).	0.7

2.3	Let r_1 and r_2 be the distances of each star from their center of mass. Find r_1 and r_2.	0.8

2.4	Let r be the distance between the stars. Find r.	0.2

3 The gravitational force is the only force acting between the stars.

3.1	Find the mass of each star up to one significant digit. The universal gravitational constant $G = 6.7\times10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$.	1.2

General Characteristics of Stars

4 Most of the stars generate energy through the same mechanism. Because of this, there is an empirical relation between their mass, M, and their luminosity, L, which is the total radiant power of the star. This relation could be written in the form $L/L_{Sun} = (M/M_{Sun})^{\alpha}$. Here, $M_{Sun} = 2.0\times10^{30}\ \mathrm{kg}$ is the solar mass and, $L_{Sun} = 3.9\times10^{26}\ \mathrm{W}$ is the solar luminosity. This relation is shown in a log-log diagram in Figure 2.

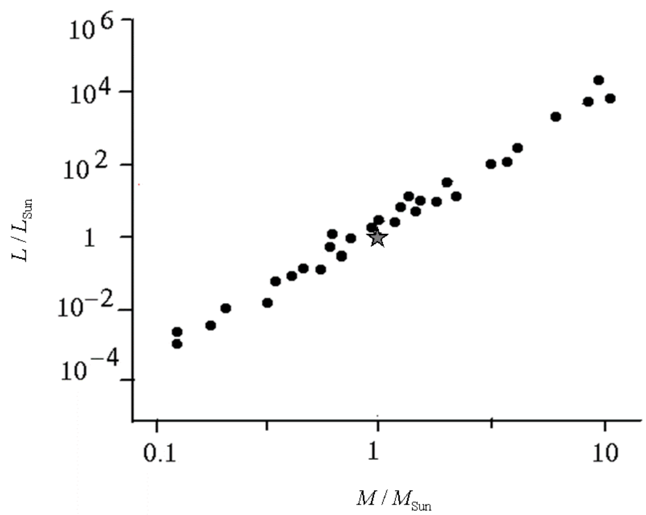

Figure 2. The luminosity of a star versus its mass varies as a power law. The diagram is log-log. The star-symbol represents Sun with a mass of $2.0\times10^{30}\ kg$ and luminosity of $3.9\times10^{26}\ \mathrm{W}$.

4.1	Find α up to one significant digit.	0.6

4.2	Let L_1 and L_2 be the luminosity of the stars in the binary system studied in the previous sections. Find L_1 and L_2.	0.6

4.3	What is the distance, d, of the star system from us in light years? To find the distance you can use the diagram of Figure 1. One light year is the distance light travels in one year.	0.9

4.4	What is the maximum angular distance, θ, between the stars from our observation point?	0.4

4.5	What is the smallest aperture size for an optical telescope, D, that can resolve these two stars?	0.4