DIFFERENTIAL THERMOMETRIC METHOD

In this problem, we use the differential thermometric method to fulfill the two following tasks:

1. Finding the temperature of solidification of a crystalline solid substance.
2. Determining the efficiency of a solar cell.

A. Differential thermometric method

In this experiment forward biased silicon diodes are used as temperature sensors to measure temperature. If the electric current through the diode is constant, then the voltage drop across the diode depends on the temperature according to the relation

$$V(T) = V(T_0) - \alpha(T - T_0) \qquad (1)$$

where $V(T)$ and $V(T_0)$ are respectively the voltage drops across the diode at temperature T and at room temperature T_0 (measured in ^{o}C), and the factor

$$\alpha = (2.00 \pm 0.03)\,\mathrm{mV/^{o}C} \qquad (2)$$

The value of $V(T_0)$ may vary slightly from diode to diode.

If two such diodes are placed at different temperatures, the difference between the temperatures can be measured from the difference of the voltage drops across the two diodes. The difference of the voltage drops, called the *differential voltage*, can be measured with high precision; hence the temperature difference can also be measured with high precision. This method is called the *differential thermometric method*. The electric circuit used with the diodes in this experiment is shown in Figure 1. Diodes D_1 and D_2 are forward biased by a 9V battery, through 10 kΩ resistors, R_1 and R_2. This circuit keeps the current in the two diodes approximately constant.

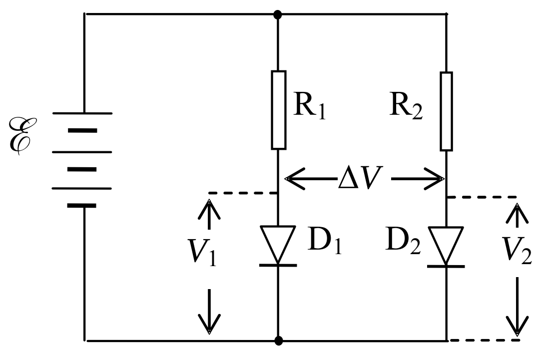


Figure 1. Electric circuit of the diode

If the temperature of diode D_1 is T_1 and that of D_2 is T_2, then according to (1), we have:

$$V_1(T_1) = V_1(T_0) - \alpha(T_1 - T_0)$$

and

$$V_2(T_2) = V_2(T_0) - \alpha(T_2 - T_0)$$

The differential voltage is:

$$\Delta V = V_2(T_2) - V_1(T_1) = V_2(T_0) - V_1(T_0) - \alpha(T_2 - T_1) = \Delta V(T_0) - \alpha(T_2 - T_1)$$

$$\Delta V = \Delta V(T_0) - \alpha \Delta T \qquad (3)$$

in which $\Delta T = T_2 - T_1$. By measuring the differential voltage ΔV, we can determine the temperature difference.

To bias the diodes, we use a circuit box, the diagram of which is shown in Figure 2.

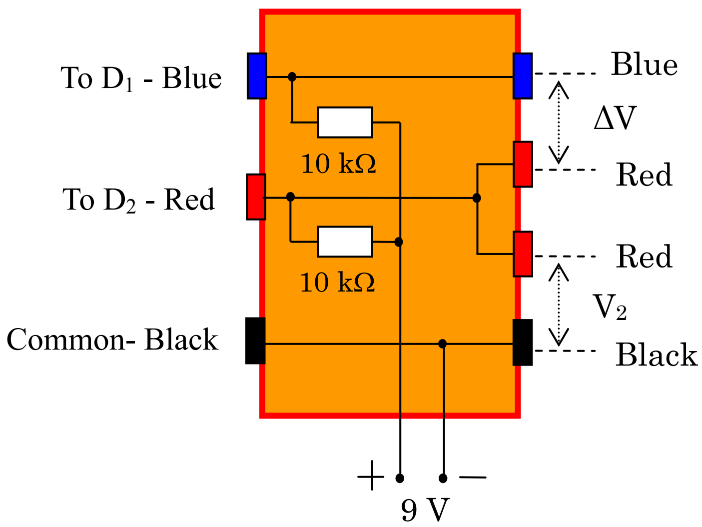


Figure 2. Diagram of the circuit box
(top view)

The circuit box contains two biasing resistors of 10 kΩ for the diodes, electrical leads to the 9 V battery, sockets for connecting to the diodes D_1 and D_2, and sockets for connecting to digital multimeters to measure the voltage drop V_2 on diode D_2 and the differential voltage ΔV of the diodes D_1 and D_2.

B. Task 1: Finding the temperature of solidification of a crystalline substance

1. Aim of the experiment

If a crystalline solid substance is heated to the melting state and then cooled down, it solidifies at a fixed temperature T_s, called *temperature of solidification*, also called the *melting point* of the substance. The *traditional method* to determine T_s is to follow the change in temperature with time during the cooling process. Due to the fact that the solidification process is accompanied by the release of the latent heat of the phase transition, the temperature of the substance does not change while the substance is solidifying. If the amount of the substance is large enough, the time interval in which the temperature remains constant is rather long, and one can easily determine this temperature. On the contrary, if the amount of substance is small, this time interval is too short to be observed and hence it is difficult to determine T_s.

In order to determine T_s in case of small amount of substance, we use the *differential thermometric method*, whose principle can be summarized as follows. We use two identical small dishes, one containing a small amount of the substance to be studied, called the *sample dish*, and the other not containing the substance, called the *reference dish*. The two dishes are put on a heat source, whose temperature varies slowly with time. The thermal flows to and from the two dishes are nearly the same. Each dish contains a temperature sensor (a forward biased silicon diode). While there is no phase change in the substance, the temperature T_{samp} of the sample dish and the temperature T_{ref} of the reference dish vary at nearly the same rate, and thus $\Delta T = T_{ref} - T_{samp}$ varies slowly with T_{samp}. If there is a phase change in the substance, and during the phase change T_{samp} does not vary and equals T_s, while T_{ref} steadily varies, then ΔT varies quickly. The plot of ΔT versus T_{samp} shows an abrupt change. The value of T_{samp} corresponding to the abrupt change of ΔT is indeed T_s.

The aim of this experiment is to determine the temperature of solidification T_s of a pure crystalline substance, having T_s in the range from 50^{o}C to 70^{o}C, by using the traditional and differential thermal analysis methods. The amount of substance used in the experiment is about 20 mg.

2. Apparatus and materials

1. The heat source is a 20 W halogen lamp.

2. The dish holder is a bakelite plate with a square hole in it. A steel plate is fixed on the hole. Two small magnets are put on the steel plate.

3. Two small steel dishes, each contains a silicon diode soldered on it. One dish is used as the reference dish, the other - as the sample dish.

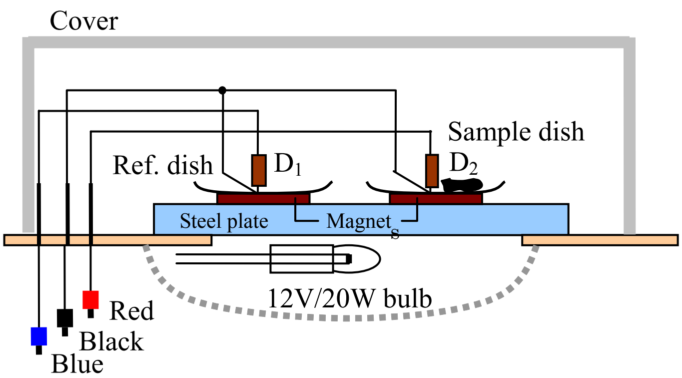


Figure 3. Apparatus for measuring the solidification temperature

Each dish is placed on a magnet. The magnetic force maintains the contact between the dish, the magnet and the steel plate. The magnets also keep a moderate thermal contact between the steel plate and the dishes.

A grey plastic box used as a cover to protect the dishes from the outside influence.

Figure 3 shows the arrangement of the dishes and the magnets on the dish holder and the light bulb.

4. Two digital multimeters are used as voltmeters. They can also measure room temperature by turning the Function selector to the "^{o}C/^{o}F" function. The voltage function of the multimeter has an error of ±2 on the last digit.

Note: to prevent the multimeter (see Figure 9) from going into the "Auto power

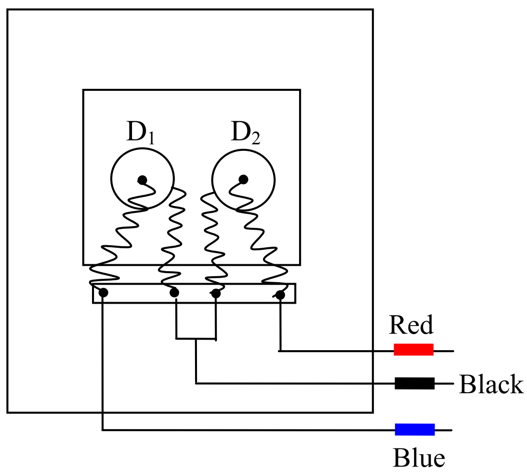


Figure 4. The dishes on the dish holder (top view)

off" function, turn the Function selector from OFF position to the desired function while pressing and holding the SELECT button.

5. A circuit box as shown in Figure 2.
6. A 9 V battery.
7. Electrical leads.
8. A small ampoule containing about 20 mg of the substance to be measured.
9. A stop watch
10. A calculator
11. Graph papers.

3. Experiment

1. The magnets are placed on two equivalent locations on the steel plate. The reference dish and the empty sample dish are put on the magnets as shown in the Figure 4. We use the dish on the left side as the reference dish, with diode D_1 on it (D_1 is called the reference diode), and the dish on the right side as the sample dish, with diode D_2 on it (D_2 is called the measuring diode).

Put the lamp-shade up side down as shown in Figure 5. Do not switch the lamp on. Put the dish holder on the lamp. Connect the apparatuses so that you can measure the voltage drop on the diode D_2, that is $V_{\text{samp}} = V_2$, and the differential voltage ΔV .

In order to eliminate errors due to the warming up period of the instruments and devices, it is strongly recommended that the complete measurement circuit be switched on for about 5 minutes before starting real experiments.

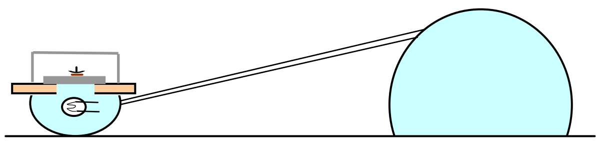


Figure 5.
Using the halogen lamp as a heat source

1.1. Measure the room temperature T_0 and the voltage drop $V_{\text{samp}}(T_0)$ across diode D_2 fixed to the sample dish, at room temperature T_0.

1.2. Calculate the voltage drops $V_{\text{samp}}(50^{\circ}\text{C})$, $V_{\text{samp}}(70^{\circ}\text{C})$ and $V_{\text{samp}}(80^{\circ}\text{C})$ on the measuring diode at temperatures 50°C, 70°C and 80°C, respectively.

2. With both dishes still empty, switch the lamp on. Follow V_{sam}. When the temperature of the sample dish reaches $T_{samp} \sim 80^oC$, switch the lamp off.

2.1. Wait until $T_{samp} \sim 70^oC$, and then follow the change in V_{samp} and ΔV with time, while the steel plate is cooling down. Note down the values of V_{samp} and ΔV every 10 s to 20 s in the table provided in the answer sheet. If ΔV varies quickly, the time interval between consecutive measurements may be shorter. When the temperature of the sample dish decreases to $T_{samp} \sim 50^oC$, the measurement is stopped.

2.2. Plot the graph of V_{samp} versus t, called Graph 1, on a graph paper provided.

2.3. Plot the graph of ΔV versus V_{samp}, called Graph 2, on a graph paper provided.

Note: for 2.2 and 2.3 do not forget to write down the correct name of each graph.

3. Pour the substance from the ampoule into the sample dish. Repeat the experiment identically as mentioned in section **2**.

3.1. Write down the data of V_{samp} and ΔV with time t in the table provided in the answer sheet.

3.2. Plot the graph of V_{samp} versus t, called Graph 3, on a graph paper provided.

3.3. Plot the graph of ΔV versus V_{samp}, called Graph 4, on a graph paper provided.

Note: for 3.2 and 3.3 do not forget to write down the correct name of each graph.

4. By comparing the graphs in section **2** and section **3**, determine the temperature of solidification of the substance.

4.1. Using the traditional method to determine T_s: by comparing the graphs of V_{samp} versus t in sections 3 and 2, i.e. Graph 3 and Graph 1, mark the point on Graph 3 where the substance solidifies and determine the value V_s (corresponding to this point) of V_{samp}.

Find out the temperature of solidification T_s of the substance and estimate its error.

4.2. Using the differential thermometric method to determine T_s: by comparing the graphs of ΔV versus V_{samp} in sections 3 and 2, i.e. Graph 4 and Graph 2**,** mark the point on Graph 4 where the substance solidifies and determine the value V_s of V_{samp}.

Find out the temperature of solidification T_s of the substance.

4.3. From errors of measurement data and instruments, calculate the error of T_s obtained with the differential thermometric method. Write down the error calculations and finally write down the values of T_s together with its error in the answer sheet.

C. Task 2: Determining the efficiency of a solar cell under illumination of an incandescent lamp

1. Aim of the experiment

The aim of the experiment is to determine the *efficiency* of a solar cell under illumination of an incandescent lamp. Efficiency is defined as the ratio of the electrical power that the solar cell can supply to an external circuit, to the total radiant power received by the cell. The efficiency depends on the incident radiation spectrum. In this experiment the radiation incident to the cell is that of an incandescent halogen lamp. In order to determine the efficiency of the solar cell, we have to measure the *irradiance* E at a point situated under the lamp, at a distance d from the lamp along the vertical direction, and the *maximum power* $P_{\max}$ of the solar cell when it is placed at this point. In this experiment, d = 12 cm (Figure 6). Irradiance E can be defined by:

$$E = \Phi / S$$

in which Φ is the radiant flux (radiant power), and S is the area of the illuminated surface.

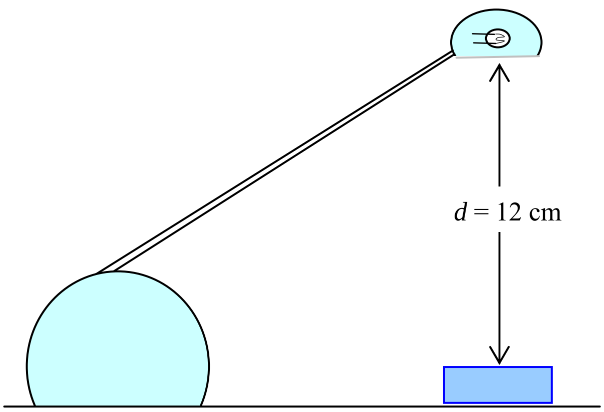


Figure 6.
Using the halogen lamp
as a light source

2. Apparatus and materials

1. The light source is a 20W halogen lamp.

2. The radiation detector is a hollow cone made of copper, the inner surface of it is blackened with soot (Figure 7). The cone is incompletely thermally isolated from the surrounding. In this experiment, the detector is considered an ideal black body. To measure temperature, we use silicon diodes. The measuring diode is fixed to the radiation detector (D_2 in Figure 1 and Figure 7), so that its temperature equals that of the cone. The reference diode is placed on the inner side of the wall of the box containing the detector; its temperature equals that of the surrounding. The total heat capacity of the detector (the cone and the measuring diode) is $C = (0.69 \pm 0.02)$ J/K. The detector is covered by a very thin polyethylene film; the radiation absorption and reflection of which can be neglected.

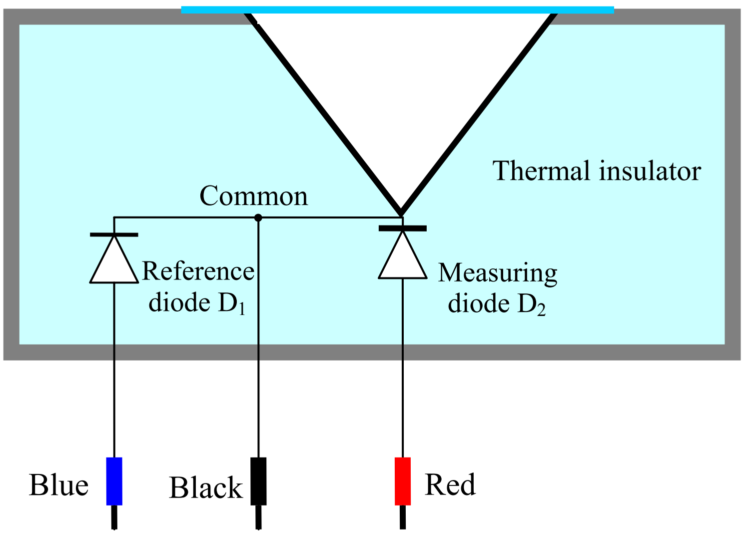


Figure 7. Diagram of the radiation detector

3. A circuit box as shown in Figure 2.

4. A piece of solar cell fixed on a plastic box (Figure 8). The area of the cell includes some metal connection strips. For the efficiency calculation these strips are considered parts of the cell.

5. Two digital multimeters. When used to measure the voltage, they have a very large internal resistance, which can be considered infinitely large. When we use them to measure the current, we cannot neglect their internal resistance. The voltage function of the multimeter has an error of ±2 on the last digit.

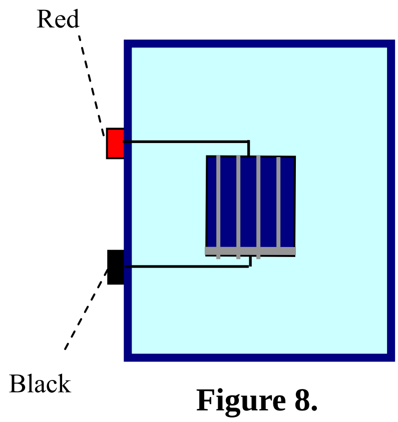


Figure 8.
The solar cell

The multimeters can also measure the room temperature.

Note: to prevent the multimeter (see Figure 9) from going into the "Auto power off" function, turn the Function selector from OFF position to the desired function while pressing and holding the SELECT button.

6. A 9 V battery
7. A variable resistor.
8. A stop watch
9. A ruler with 1mm divisions
10. Electrical leads.
11. Graph papers.

3. Experiment

When the detector receives energy from radiation, it heats up. At the same time, the detector loses its heat by several mechanisms, such as thermal conduction, convection, radiation etc...Thus, the radiant energy received by detector in a time interval dt is equal to the sum of the energy needed to increase the detector temperature and the energy transferred from the detector to the surrounding:

$$\Phi dt = CdT + dQ$$

where C is the heat capacity of the detector and the diode, dT - the temperature increase and dQ - the heat loss.

When the temperature difference between the detector and the surrounding $\Delta T = T - T_0$ is small, we can consider that the heat dQ transferred from the detector to the surrounding in the time interval dt is approximately proportional to ΔT and dt, that is $dQ = k\Delta T dt$, with k being a factor having the dimension of W/K. Hence, assuming that k is constant and ΔT is small, we have:

$$\Phi dt = CdT + k\Delta T dt = Cd(\Delta T) + k\Delta T dt$$

or

$$\frac{d(\Delta T)}{dt} + \frac{k}{C}\Delta T = \frac{\Phi}{C} \qquad (4)$$

The solution of this differential equation determines the variation of the temperature difference ΔT with time t, from the moment the detector begins to receive the light with a constant irradiation, assuming that at t=0, ΔT =0

$$\Delta T(t) = \frac{\Phi}{k}\left(1 - e^{-\frac{k}{C}t}\right) \qquad (5)$$

When the radiation is switched off, the mentioned above differential equation becomes

$$\frac{d(\Delta T)}{dt} + \frac{k}{C}\Delta T = 0 \qquad (6)$$

and the temperature difference ΔT varies with the time according to the following formula:

$$\Delta T\,(t) = \Delta T\,(0)\,e^{-\frac{k}{C}t} \qquad (7)$$

where $\Delta T(0)$ is the temperature difference at $t = 0$ (the moment when the measurement starts).

1. Determine the room temperature T_0.

2. Compose an electric circuit comprising the diode sensors, the circuit box and the multimeters to measure the temperature of the detector.

In order to eliminate errors due to the warming up period of the instruments and devices, it is strongly recommended that the complete measurement circuit be switched on for about 5 minutes before starting real experiments.

2.1. Place the detector under the light source, at a distance of d = 12 cm to the lamp. The lamp is off. Follow the variation of ΔV for about 2 minutes with sampling intervals of 10 s and determine the value of $\Delta V(T_0)$ in equation (3).

2.2. Switch the lamp on to illuminate the detector. Follow the variation of ΔV. Every 10-15 s, write down a value of ΔV in the table provided in the answer sheet. (Note: columns x and y of the table will be used later in section **4.**). After 2 minutes, switch the lamp off.

2.3. Move the detector away from the lamp. Follow the variation of ΔV for about 2 minutes after that. Every 10-15 s, write down a value of ΔV in the table provided in the answer sheet. (Note: columns x and y of the table will be used later in section **3.**).

Hints: *As the detector has a thermal inertia, it is recommended not to use some data obtained immediately after the moment the detector begins to be illuminated or ceases to be illuminated.*

3. Plot a graph in an x-y system of coordinates, with variables x and y chosen appropriately, in order to prove that after the lamp is switched off, equation (7) is satisfied.

3.1. Write down the expression for variables x and y.

3.2. Plot a graph of y versus x, called Graph 5.

3.3. From the graph, determine the value of k.

4. Plot a graph in an x-y system of coordinates, with variables x and y chosen appropriately, in order to prove that when the detector is illuminated, equation (5) is satisfied.

4.1. Write down the expressions for variables x and y.

4.2. Plot a graph of y versus x, called Graph 6.

4.3. Determine the irradiance E at the orifice of the detector.

5. Put the solar cell to the same place where the radiation detector was. Connect the solar cell to an appropriate electric circuit comprising the multimeters and a variable resistor which is used to change the load of the cell. Measure the current in the circuit and the voltage on the cell at different values of the resistor.

5.1. Draw a diagram of the circuit used in this experiment.

5.2. By rotating the knob of the variable resistor, you change the value of the load. Note the values of current I and voltage V at each position of the knob.

5.3. Plot a graph of the power of the cell, which supplies to the load, as a function of the current through the cell. This is Graph 7.

5.4. From the graph deduce the maximum power P_{max} of the cell and estimate its error.

5.5. Write down the expression for the efficiency of the cell that corresponds to the obtained maximum power. Calculate its value and error.

Contents of the experiment kit (see also Figure 10)

1	Halogen lamp 220 V/ 20 W	9	Stop watch
2	Dish holder	10	Calculator
3	Dish	11	Radiation detector
4	Multimeter	12	Solar cell
5	Circuit box	13	Variable resistor
6	9 V battery	14	Ruler
7	Electrical leads	15	Box used as a cover
8	Ampoule with substance to be measured		

Note: to prevent the multimeter (see Figure 9) from going into the "Auto power off" function, turn the Function selector from OFF position to the desired function while pressing and holding the SELECT button.

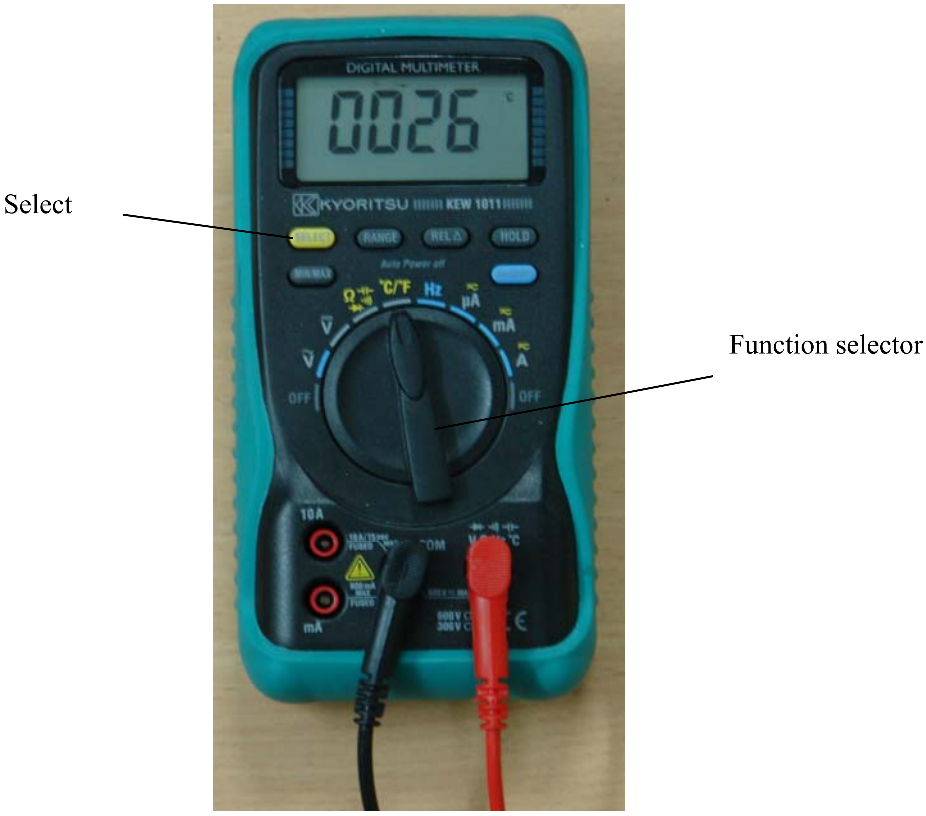


Figure 9. Digital multimeter

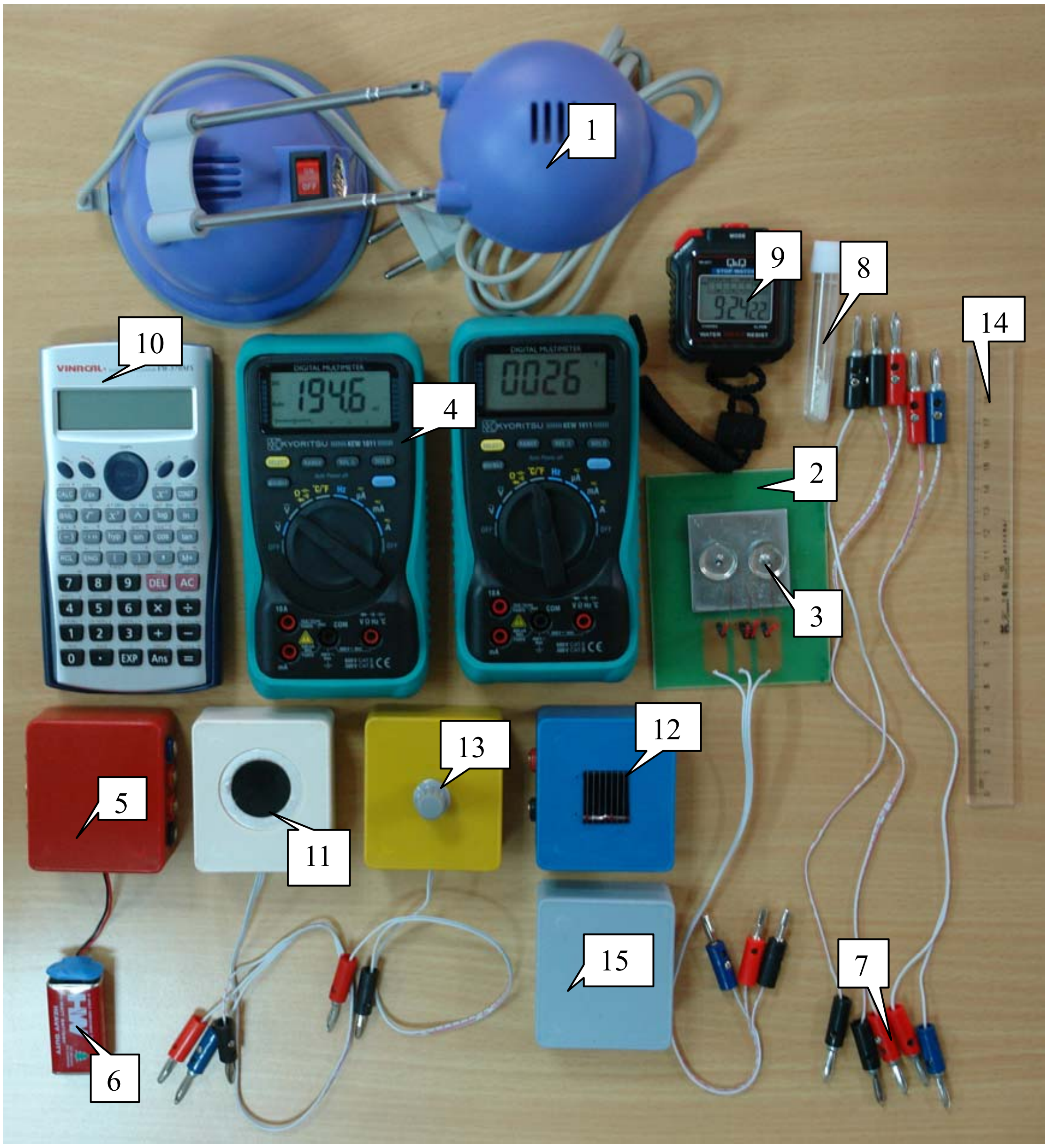


Figure 10. Contents of the experiment kit

WATER-POWERED RICE-POUNDING MORTAR

A. Introduction

Rice is the main staple food of most people in Vietnam. To make white rice from paddy rice, one needs separate of the husk (a process called "hulling") and separate the bran layer ("milling"). The hilly parts of northern Vietnam are abundant with water streams, and people living there use *water-powered rice-pounding mortar* for bran layer separation. Figure 1 shows one of such mortars., Figure 2 shows how it works.

B. Design and operation

1. Design.

The rice-pounding mortar shown in Figure 1 has the following parts:

The mortar, basically a wooden container for rice.

The lever, which is a tree trunk with one larger end and one smaller end. It can rotate around a horizontal axis. A *pestle* is attached perpendicularly to the lever at the smaller end. The length of the pestle is such that it touches the rice in the mortar when the lever lies horizontally. The larger end of the lever is carved hollow to form a bucket. The shape of the bucket is crucial for the mortar's operation.

2. Modes of operation

The mortar has two modes.

Working mode. In this mode, the mortar goes through an operation cycle illustrated in Figure 2.

The rice-pounding function comes from the work that is transferred from the pestle to the rice during stage f) of Figure 2. If, for some reason, the pestle never touches the rice, we say that the mortar is not working.

*Rest mode with the lever lifted up***.** During stage c) of the operation cycle (Figure 2), as the tilt angle α increases, the amount of water in the bucket decreases. At one particular moment in time, the amount of water is just enough to counterbalance the weight of the lever. Denote the tilting angle at this instant by β. If the lever is kept at angle β and the initial angular velocity is zero, then the lever will remain at this position forever. This is the rest mode with the lever lifted up. The stability of this position depends on the flow rate of water into the bucket, Φ. If Φ exceeds some value Φ_2, then this rest mode is stable, and the mortar cannot be in the working mode.

In other words, Φ_2 is the minimal flow rate for the mortar not to work.

Figure 1
A water-powered rice-pounding mortar

OPERATION CYCLE OF A WATER-POWERED RICE-POUNDING MORTAR

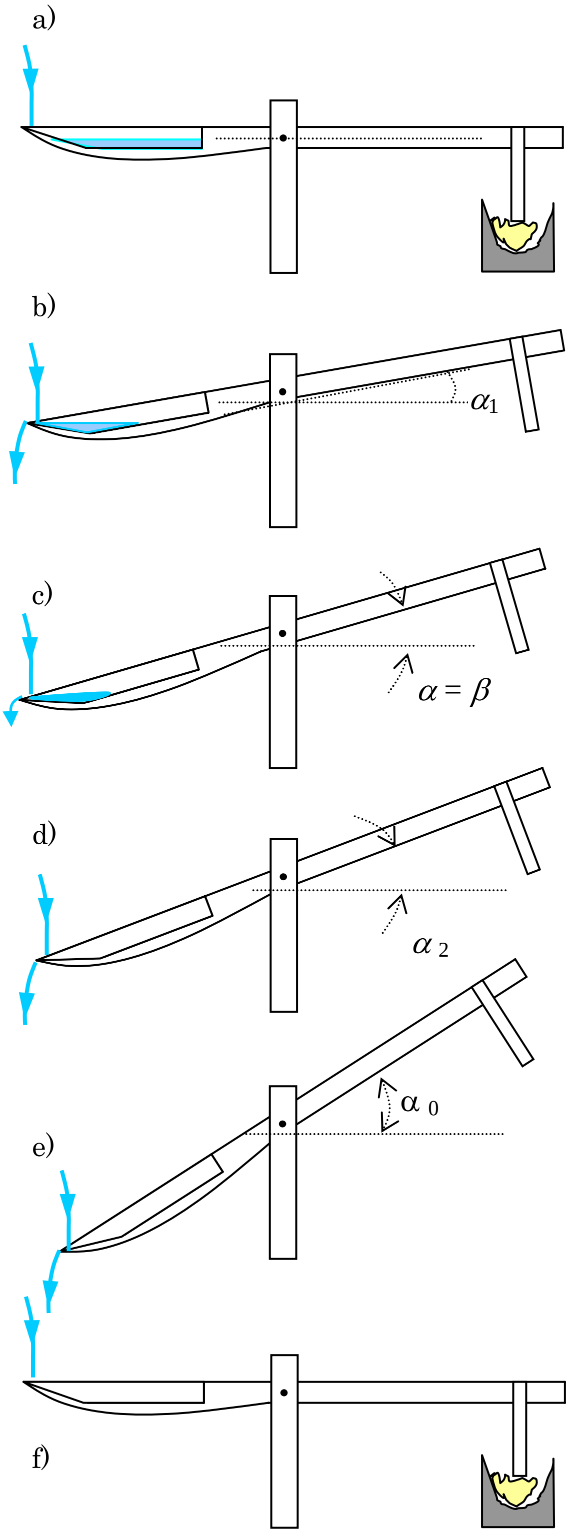


Figure 2

a) At the beginning there is no water in the bucket, the pestle rests on the mortar. Water flows into the bucket with a small rate, but for some time the lever remains in the horizontal position.

b) At some moment the amount of water is enough to lift the lever up. Due to the tilt, water rushes to the farther side of the bucket, tilting the lever more quickly. Water starts to flow out at $\alpha = \alpha_1$.

c) As the angle α increases, water starts to flow out. At some particular tilt angle, $\alpha = \beta$, the total torque is zero.

d) α continues increasing, water continues to flow out until no water remains in the bucket.

e) α keeps increasing because of inertia. Due to the shape of the bucket, water falls into the bucket but immediately flows out. The inertial motion of the lever continues until α reaches the maximal value α_0.

f) With no water in the bucket, the weight of the lever pulls it back to the initial horizontal position. The pestle gives the mortar (with rice inside) a pound and a new cycle begins.

C. The problem

Consider a water-powered rice-pounding mortar with the following parameters (Figure 3)

The mass of the lever (including the pestle but without water) is $M = 30$ kg,

The center of mass of the lever is G.. The lever rotates around the axis T (projected onto the point T on the figure).

The moment of inertia of the lever around T is $I = 12\ \mathrm{kg \cdot m^2}$.

When there is water in the bucket, the mass of water is denoted as m, the center of mass of the water body is denoted as N.

The tilt angle of the lever with respect to the horizontal axis is α.

The main length measurements of the mortar and the bucket are as in Figure 3.

Neglect friction at the rotation axis and the force due to water falling onto the bucket. In this problem, we make an approximation that the water surface is always horizontal.

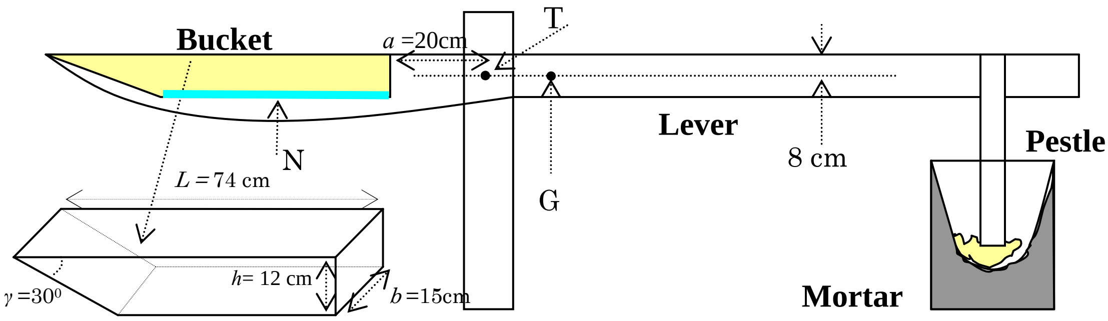


Figure 3 Design and dimensions of the rice-pounding mortar

1. The structure of the mortar

At the beginning, the bucket is empty, and the lever lies horizontally. Then water flows into the bucket until the lever starts rotating. The amount of water in the bucket at this moment is $m = 1.0$ kg.

1.1. Determine the distance from the center of mass G of the lever to the rotation axis T. It is known that GT is horizontal when the bucket is empty.

1.2. Water starts flowing out of the bucket when the angle between the lever and the horizontal axis reaches α_1. The bucket is completely empty when this angle is α_2. Determine α_1 and α_2.

1.3. Let $\mu(\alpha)$ be the total torque (relative to the axis T) which comes from the weight of the lever and the water in the bucket. $\mu(\alpha)$ is zero when $\alpha = \beta$. Determine β and the mass m_1 of water in the bucket at this instant.

2. Parameters of the working mode

Let water flow into the bucket with a flow rate Φ which is constant and small. The amount of water flowing into the bucket when the lever is in motion is negligible. In this part, neglect the change of the moment of inertia during the working cycle.

2.1. Sketch a graph of the torque μ as a function of the angle α, $\mu(\alpha)$, during one operation cycle. Write down explicitly the values of $\mu(\alpha)$ at angle α_1, α_2, and $\alpha = 0$.

2.2. From the graph found in section 2.1., discuss and give the geometric interpretation of the value of the total energy W_{total} produced by $\mu(\alpha)$ and the work W_{pounding} that is transferred from the pestle to the rice.

2.3. From the graph representing μ versus α, estimate α_0 and W_{pounding} (assume the kinetic energy of water flowing into the bucket and out of the bucket is negligible.) You may replace curve lines by zigzag lines, if it simplifies the calculation.

3. The rest mode

Let water flow into the bucket with a constant rate Φ, but one cannot neglect the amount of water flowing into the bucket during the motion of the lever.

3.1. Assuming the bucket is always overflown with water,

3.1.1. Sketch a graph of the torque μ as a function of the angle α in the vicinity of $\alpha = \beta$. To which kind of equilibrium does the position $\alpha = \beta$ of the lever belong?

3.1.2. Find the analytic form of the torque $\mu(\alpha)$ as a function of $\Delta\alpha$ when $\alpha = \beta + \Delta\alpha$, and $\Delta\alpha$ is small.

3.1.3. Write down the equation of motion of the lever, which moves with zero initial velocity from the position $\alpha = \beta + \Delta\alpha$ ($\Delta\alpha$ is small). Show that the motion is, with good accuracy, harmonic oscillation. Compute the period τ.

3.2. At a given Φ, the bucket is overflown with water at all times only if the lever moves sufficiently slowly. There is an upper limit on the amplitude of harmonic oscillation, which depends on Φ. Determine the minimal value Φ_1 of Φ (in kg/s) so that the lever can make a harmonic oscillator motion with amplitude 1^o.

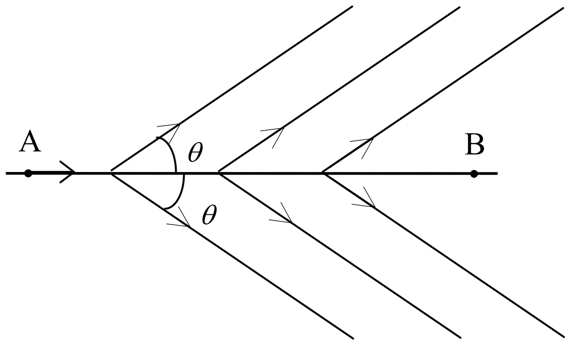
3.3. Assume that Φ is sufficiently large so that during the free motion of the lever when the tilting angle decreases from α_2 to α_1 the bucket is always overflown with water. However, if Φ is too large the mortar cannot operate. Assuming that the motion of the lever is that of a harmonic oscillator, estimate the minimal flow rate Φ_2 for the rice-pounding mortar to not work.

CHERENKOV LIGHT AND RING IMAGING COUNTER

Light propagates in vacuum with the speed c. There is no particle which moves with a speed higher than c. However, it is possible that in a transparent medium a particle moves with a speed v higher than the speed of the light in the same medium $\frac{c}{n}$, where n is the refraction index of the medium. Experiment (Cherenkov, 1934) and theory (Tamm and Frank, 1937) showed that a charged particle, moving with a speed v in a transparent medium with refractive index n such that $v > \frac{c}{n}$, radiates light, called *Cherenkov light*, in directions forming with the trajectory an angle

$$\theta = \mathbf{arccos}\frac{1}{\beta n} \qquad (1)$$

where $\beta = \frac{v}{c}$.



1. To establish this fact, consider a particle moving at constant velocity $v > \frac{c}{n}$ on a straight line. It passes A at time 0 and B at time t_1. As the problem is symmetric with respect to rotations around AB, it is sufficient to consider light rays in a plane containing AB.

At any point C between A and B, the particle emits a spherical light wave, which propagates with velocity $\frac{c}{n}$. We define the wave front at a given time t as the envelope of all these spheres at this time.

1.1. Determine the wave front at time t_1 and draw its intersection with a plane containing the trajectory of the particle.

1.2. Express the angle φ between this intersection and the trajectory of the particle in terms of n and β.

2. Let us consider a beam of particles moving with velocity $v > \frac{c}{n}$, such that the angle θ is small, along a straight line IS. The beam crosses a concave spherical mirror of focal length f and center C, at point S. SC makes with SI a small angle α (see the figure in the Answer Sheet). The particle beam creates a ring image in the focal plane of the mirror.

Explain why with the help of a sketch illustrating this fact. Give the position of the center O and the radius r of the ring image.

This set up is used in *ring imaging Cherenkov counters* (RICH) and the medium which the particle traverses is called the *radiator*.

Note: in all questions of the present problem, terms of second order and higher in α and θ will be neglected.

3. A beam of particles of known momentum $p = 10.0\ \text{GeV}/c$ consists of three types of particles: protons, kaons and pions, with rest mass $M_{\text{p}} = 0.94\ \text{GeV}/c^2$, $M_{\kappa} = 0.50\ \text{GeV}/c^2$ and $M_{\pi} = 0.14\ \text{GeV}/c^2$, respectively. Remember that pc and Mc^2 have the dimension of an energy, and 1 eV is the energy acquired by an electron after being accelerated by a voltage 1 V, and 1 GeV = 10^9 eV, 1 MeV = 10^6 eV.

The particle beam traverses an air medium (the radiator) under the pressure P. The refraction index of air depends on the air pressure P according to the relation $n = 1 + aP$ where $a = 2.7\times10^{-4}\ \text{atm}^{-1}$

3.1. Calculate for each of the three particle types the minimal value $P_{\min}$ of the air pressure such that they emit Cherenkov light.

3.2. Calculate the pressure $P_{\frac{1}{2}}$ such that the ring image of kaons has a radius equal to one half of that corresponding to pions. Calculate the values of θ_{κ} and θ_{π} in this case.

Is it possible to observe the ring image of protons under this pressure?

4. Assume now that the beam is not perfectly monochromatic: the particles momenta are distributed over an interval centered at $10\ \text{GeV}/c$ having a half width at half height Δp. This makes the ring image broaden, correspondingly θ distribution has a half width at half height $\Delta\theta$. The pressure of the radiator is $P_{\frac{1}{2}}$ determined in 3.2.

4.1. Calculate $\dfrac{\Delta\theta_{\kappa}}{\Delta p}$ and $\dfrac{\Delta\theta_{\pi}}{\Delta p}$, the values taken by $\dfrac{\Delta\theta}{\Delta p}$ in the pions and kaons cases.

4.2. When the separation between the two ring images, $\theta_{\pi} - \theta_{\kappa}$, is greater than 10 times the half-width sum $\Delta\theta = \Delta\theta_\kappa + \Delta\theta_\pi$, that is $\theta_\pi - \theta_\kappa > 10\ \Delta\theta$, it is possible to distinguish well the two ring images. Calculate the maximal value of Δp such that the two ring images can still be well distinguished.

5. Cherenkov first discovered the effect bearing his name when he was observing a bottle of water located near a radioactive source. He saw that the water in the bottle emitted light.

5.1. Find out the minimal kinetic energy $T_{\min}$ of a particle with a rest mass M moving in water, such that it emits Cherenkov light. The index of refraction of water is $n = 1.33$.

5.2. The radioactive source used by Cherenkov emits either α particles (i.e. helium nuclei) having a rest mass $M_\alpha = 3.8\ \mathrm{GeV}/c^2$ or β particles (i.e. electrons) having a rest mass $M_\mathrm{e} = 0.51\ \mathrm{MeV}/c^2$. Calculate the numerical values of $T_{\min}$ for α particles and β particles.

Knowing that the kinetic energy of particles emitted by radioactive sources never exceeds a few MeV, find out which particles give rise to the radiation observed by Cherenkov.

6. In the previous sections of the problem, the dependence of the Cherenkov effect on wavelength λ has been ignored. We now take into account the fact that the Cherenkov radiation of a particle has a broad continuous spectrum including the visible range (wavelengths from 0.4 μm to 0.8 μm). We know also that the index of refraction n of the radiator decreases linearly by 2% of $n-1$ when λ increases over this range.

6.1. Consider a beam of pions with definite momentum of $10.0\ \mathrm{GeV}/c$ moving in air at pressure 6 atm. Find out the angular difference $\delta\theta$ associated with the two ends of the visible range.

6.2. On this basis, study qualitatively the effect of the dispersion on the ring image of pions with momentum distributed over an interval centered at $p = 10\ \mathrm{GeV}/c$ and having a half width at half height $\Delta p = 0.3\ \mathrm{GeV}/c$.

6.2.1. Calculate the broadening due to dispersion (varying refraction index) and that due to achromaticity of the beam (varying momentum).

6.2.2. Describe how the color of the ring changes when going from its inner to outer edges by checking the appropriate boxes in the Answer Sheet.

CHANGE OF AIR TEMPERATURE WITH ALTITUDE, ATMOSPHERIC STABILITY AND AIR POLLUTION

Vertical motion of air governs many atmospheric processes, such as the formation of clouds and precipitation and the dispersal of air pollutants. If the atmosphere is *stable*, vertical motion is restricted and air pollutants tend to be accumulated around the emission site rather than dispersed and diluted. Meanwhile, in an *unstable* atmosphere, vertical motion of air encourages the vertical dispersal of air pollutants. Therefore, the pollutants' concentrations depend not only on the strength of emission sources but also on the *stability* of the atmosphere.

We shall determine the atmospheric stability by using the concept of *air parcel* in meteorology and compare the temperature of the air parcel rising or sinking adiabatically in the atmosphere to that of the surrounding air. We will see that in many cases an air parcel containing air pollutants and rising from the ground will come to rest at a certain altitude, called a *mixing height*. The greater the mixing height, the lower the air pollutant concentration. We will evaluate the mixing height and the concentration of carbon monoxide emitted by motorbikes in the Hanoi metropolitan area for a morning rush hour scenario, in which the vertical mixing is restricted due to a temperature inversion (air temperature increases with altitude) at elevations above 119 m.

Let us consider the air as an ideal diatomic gas, with molar mass μ = 29 g/mol.

Quasi equilibrium adiabatic transformation obey the equation $pV^{\gamma} = \text{const}$, where $\gamma = \dfrac{c_p}{c_V}$ is the ratio between isobaric and isochoric heat capacities of the gas.

The student may use the following data if necessary:

The universal gas constant is $R = 8.31$ J/(mol.K).

The atmospheric pressure on ground is $p_0 = 101.3$ kPa

The acceleration due to gravity is constant, $g = 9.81$ m/s^2

The molar isobaric heat capacity is $c_p = \dfrac{7}{2}R$ for air.

The molar isochoric heat capacity is $c_V = \dfrac{5}{2}R$ for air.

Mathematical hints

a. $\int \frac{dx}{A+Bx} = \frac{1}{B}\int \frac{d\left(A+Bx\right)}{A+Bx} = \frac{1}{B}\mathbf{ln}\left(A+Bx\right)$

b. The solution of the differential equation $\frac{dx}{dt} + Ax = B$ (with A and B constant) is $x\left(t\right) = x_1\left(t\right) + \frac{B}{A}$ where $x_1\left(t\right)$ is the solution of the differential equation $\frac{dx}{dt} + Ax = 0$.

c. $\mathbf{lim}_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

1. Change of pressure with altitude.

1.1. Assume that the temperature of the atmosphere is uniform and equal to T_0. Write down the expression giving the atmospheric pressure p as a function of the altitude z.

1.2. Assume that the temperature of the atmosphere varies with the altitude according to the relation

$$T\left(z\right) = T\left(0\right) - \Lambda z$$

where Λ is a constant, called the *temperature lapse rate* of the atmosphere (the vertical gradient of temperature is - Λ).

1.2.1. Write down the expression giving the atmospheric pressure p as a function of the altitude z.

1.2.2. A process called free convection occurs when the air density increases with altitude. At which values of Λ does the free convection occur?

2. Change of the temperature of an air parcel in vertical motion

Consider an air parcel moving upward and downward in the atmosphere. An air parcel is a body of air of sufficient dimension, several meters across, to be treated as an independent thermodynamical entity, yet small enough for its temperature to be considered uniform. The vertical motion of an air parcel can be treated as a quasi adiabatic process, i.e. the exchange of heat with the surrounding air is negligible. If the air parcel rises in the atmosphere, it expands and cools. Conversely, if it moves downward, the increasing outside pressure will compress the air inside the parcel and its temperature will increase.

As the size of the parcel is not large, the atmospheric pressure at different points on the parcel boundary can be considered to have the same value $p(z)$, with z - the altitude of the parcel center. The temperature in the parcel is uniform and equals to $T_{\text{parcel}}(z)$, which is generally different from the temperature of the surrounding air $T(z)$. In parts 2.1 and 2.2, we do not make any assumption about the form of $T(z)$.

2.1. The change of the parcel temperature T_{parcel} with altitude is defined by $\frac{dT_{\text{parcel}}}{dz} = -G$. Derive the expression of $G(T, T_{\text{parcel}})$.

2.2. Consider a special atmospheric condition in which at any altitude z the temperature T of the atmosphere equals to that of the parcel T_{parcel}, $T(z) = T_{\text{parcel}}(z)$. We use Γ to denote the value of G when $T = T_{\text{parcel}}$, that is $\Gamma = -\frac{dT_{\text{parcel}}}{dz}$ (with $T = T_{\text{parcel}}$). Γ is called *dry adiabatic lapse rate*.

2.2.1. Derive the expression of Γ

2.2.2. Calculate the numerical value of Γ.

2.2.3. Derive the expression of the atmospheric temperature $T(z)$ as a function of the altitude.

2.3. Assume that the atmospheric temperature depends on altitude according to the relation $T(z) = T(0) - \Lambda z$, where Λ is a constant. Find the dependence of the parcel temperature $T_{\text{parcel}}(z)$ on altitude z.

2.4. Write down the approximate expression of $T_{\text{parcel}}(z)$ when $|\Lambda z| << T(0)$ and $T(0) \approx T_{\text{parcel}}(0)$.

3. The atmospheric stability.

In this part, we assume that T changes linearly with altitude.

3.1. Consider an air parcel initially in equilibrium with its surrounding air at altitude

z_0, i.e. it has the same temperature $T(z_0)$ as that of the surrounding air. If the parcel is moved slightly up and down (e.g. by atmospheric turbulence), one of the three following cases may occur:

- The air parcel finds its way back to the original altitude z_0, the equilibrium of the parcel is stable. The atmosphere is said to be stable.
- The parcel keeps moving in the original direction, the equilibrium of the parcel is unstable. The atmosphere is unstable.
- The air parcel remains at its new position, the equilibrium of the parcel is indifferent. The atmosphere is said to be neutral.

What is the condition on Λ for the atmosphere to be stable, unstable or neutral?

3.2. A parcel has its temperature on ground $T_{\text{parcel}}(0)$ higher than the temperature $T(0)$ of the surrounding air. The buoyancy force will make the parcel rise. Derive the expression for the maximal altitude the parcel can reach in the case of a stable atmosphere in terms of Λ and Γ.

4. The mixing height

4.1. Table 1 shows air temperatures recorded by a radio sounding balloon at 7: 00 am on a November day in Hanoi. The change of temperature with altitude can be approximately described by the formula $T(z) = T(0) - \Lambda z$ with different lapse rates Λ in the three layers $0 < z < 96$ m, 96 m $< z <$ 119 m and 119 m$< z <$ 215 m.

Consider an air parcel with temperature $T_{\text{parcel}}(0) = 22^{\circ}$C ascending from ground. On the basis of the data given in Table 1 and using the above linear approximation, calculate the temperature of the parcel at the altitudes of 96 m and 119 m.

4.2. Determine the maximal elevation H the parcel can reach, and the temperature $T_{\text{parcel}}(H)$ of the parcel.

H is called the mixing height. Air pollutants emitted from ground can mix with the air in the atmosphere (e.g. by wind, turbulence and dispersion) and become diluted within this layer.

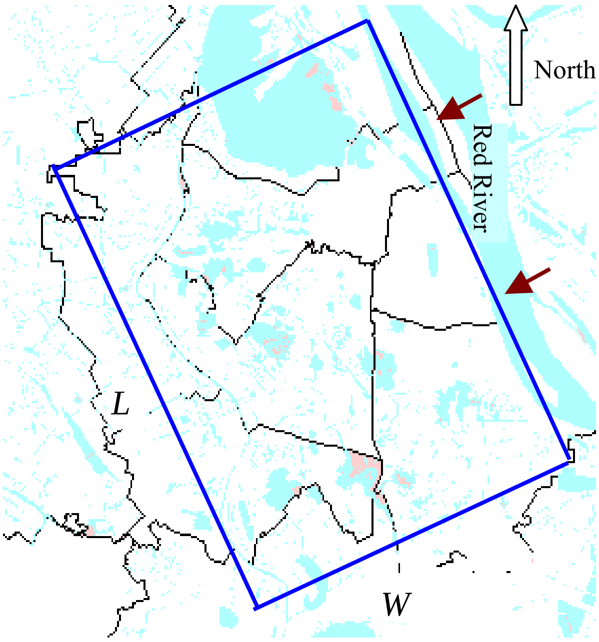
Table 1

Data recorded by a radio sounding balloon at 7:00 am on a November day in Hanoi.

Altitude, m	Temperature, °C
5	21.5
60	20.6
64	20.5
69	20.5
75	20.4
81	20.3
90	20.2
96	20.1
102	20.1
109	20.1
113	20.1
119	20.1
128	20.2
136	20.3
145	20.4
153	20.5
159	20.6
168	20.8
178	21.0
189	21.5
202	21.8
215	22.0
225	22.1
234	22.2
246	22.3
257	22.3

5. Estimation of carbon monoxide (CO) pollution during a morning motorbike rush hour in Hanoi.

Hanoi metropolitan area can be approximated by a rectangle with base dimensions L and W as shown in the figure, with one side taken along the south-west bank of the Red River.



It is estimated that during the morning rush hour, from 7:00 am to 8:00 am, there are $8x10^5$ motorbikes on the road, each running on average 5 km and emitting 12 g of CO per kilometer. The amount of CO pollutant is approximately considered as emitted uniformly in time, at a constant rate *M* during the rush hour. At the same time, the clean north-east wind blows perpendicularly to the Red River (i.e. perpendicularly to the sides *L* of the rectangle) with velocity *u*, passes the city with the same velocity, and carries a part of the CO-polluted air out of the city atmosphere.

Also, we use the following rough approximate model:

- The CO spreads quickly throughout the entire volume of the mixing layer above the Hanoi metropolitan area, so that the concentration $C(t)$ of CO at time t can be assumed to be constant throughout that rectangular box of dimensions *L*, *W* and *H*.
- The upwind air entering the box is clean and no pollution is assumed to be lost from the box through the sides parallel to the wind.
- Before 7:00 am, the CO concentration in the atmosphere is negligible.

5.1. Derive the differential equation determining the CO pollutant concentration $C(t)$ as a function of time.

5.2. Write down the solution of that equation for $C(t)$.

5.3. Calculate the numerical value of the concentration $C(t)$ at 8:00 a.m.

Given $L = 15$ km, $W = 8$ km, $u = 1$ m/s.